A Conversation on Professional Norms in Mathematics

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We are greatly indebted to the presenters at a workshop by the same name
held at Johns Hopkins University in September 2019,
without whom this conversation would not exist:

Eugenia Cheng, “Inclusion-exclusion in mathematics and beyond: who stays in, who falls out, why it happens, and what we could do about it”

Alexander Diaz-Lopez, “Becoming a better version of ourselves”

Pamela E. Harris, “Avoiding the academic savior complex: How to mentor underrepresented faculty”

Denis R. Hirschfeldt, “Mathematicians, Collective Bargaining, and the Corporatized Academy”

Mike Hill, “Queer spheres”

Dagan Karp, “An Introduction to Critical Theory in Postsecondary Mathematics Education”

Oliver Knill, “On parameters for communicating mathematical ideas”

David Kung, “From Teaching Math to Teaching Students: Transforming classroom norms among college math instructors”

Izabella Łaba, “Rethinking universities in the era of climate change”

Luis A. Leyva, “Racialized and Gendered Mechanisms of Pre-calculus and Calculus Instruction: A Window into Cultural and Professional Norms in Mathematics”

Michelle Manes, “Be the change you want to see in mathematics”

Adriana Salerno, “The mathematician as public intellectual”

Francis Su, “The College/University Divide: How Do We Fix It?”

Aris Winger, “Acknowledgement of the Impact Race has in our Practice”

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Preface

> *Being aware of how we interact and the types of relationships and communities we build are critical in order for the mathematical sciences to flourish.* —American Mathematical Society[1]

The past year has renewed a large-scale, long overdue conversation in the American mathematical community to reimagine strategies for teaching and learning, to reconsider the role of mathematics in society, and to reckon with its complicity in racial injustice. In this volume, we focus on what we can do as members of our mathematics community, assessing and reassessing some of our often unstated professional norms. Norms are local: they are how individuals interact with each other and how individuals act in an institution. They control everything from how one engages with seminar speakers to how students, postdocs, and junior faculty are mentored to who actually does what work. What can we as mathematicians and educators do in the classroom? In our departments? In our institutions? In our field?

Before it becomes possible to imagine professional norms that make mathematics more just and inclusive, we must recognize the structural forces that have contributed to the current status quo. The faculty and administrators who have set the tone in most departments have succeeded in this system, knowing how to play the game and intuitively understanding the rules. The sieves that permit only the lucky few to advance to each subsequent career stage contribute to a narrower scope of experience and understanding. The factors that have helped some succeed should be scrutinized in tandem with the obstacles that have hindered the success of others.

In this collection, we have centered the personal voices of mathematicians whose lived experiences warn against an attenuated view of mathematical talent and injurious "mentoring" practices (see especially Chapters 1 and 2 in this volume). The authors point out the paradox of promotion systems that award individual achievement in a world where research progress is most commonly made by collective effort. They also share stories about painful experiences we often do not hear.

The volume continues with concrete suggestions about how to start to do the work to build communities in which all mathematicians can flourish. The contributing authors explore what we can do in the classroom and in our public lectures, centering the humanity of our students and audience (see especially Chapters 3 and 4). They describe what we can do in our institutions, like showing solidarity with contingent teaching faculty, supporting organized labor, and confronting sexual harassment (see especially Chapters 5, 8, and 9). They discuss what we can do nationally and internationally, acknowledging structural changes in academia and the impact of our approaches to the environment. They also encourage us to draw on the skills developed through

[1] `http://www.ams.org/programs/diversity/diversity`

mathematics in order to engage with work in other disciplines, using mathematics as a language to help further understanding (see Chapters 6 and 7).

In this kind of work, we see a local-to-global phenomenon quite clearly: our work at the local level building community and creating brave and welcoming spaces glues to the work of our colleagues at other institutions, creating a systemic awareness and change across the mathematical landscape. Building open and supportive communities is hard and requires continuous effort, and facing this and figuring out how to even start can be daunting. Our goal is to provide a place where mathematicians can begin educating themselves and start taking steps to build a better, more inclusive community.

As mathematicians, we like to see ourselves as participating in a very long conversation, engaging directly with the mathematical work and mathematicians from years, decades, and centuries before. This volume is timely, reflecting the current state of discourse, but the issues discussed are timeless.

Mathilde Gerbelli-Gauthier
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Michael A. Hill
Dagan Karp
Emily Riehl
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Introduction

Emily Riehl

The theory

I am one of those lucky mathematicians who fell in love at a young age, thanks to supportive parents and excellent public schools. I chose my college knowing I wanted to be a math major, and ultimately a mathematics professor, and arrived in class on the first day knowing that "∀" abbreviates "for all" while "∃" stands for "there exists." Better yet, I'd met half of my classmates already on the high school science fair circuit and even knew the rudiments of LaTeX.

During the inevitable crises of confidence that befall any aspiring mathematician, I was bolstered by a robust support system of mathematical mentors I'd encountered along the way. The first person I came out to (via email) while worrying that my pending disclosure would ruin my relationship with my college roommates was a faculty member from the summer math camp I'd attended a few years prior. A college alumni interviewer—now a close colleague in my field, but who at that point I'd only spoken to on that one phone call—insisted that I get in touch if I ever considered switching subjects. So even on the occasions when I got totally lost during a lecture, I still felt like I belonged in the classroom.

When Piper H's 2016 Princeton PhD thesis hit the web, I was in my first semester of a tenure-track job. Any attempt to summarize H's unique discursive contributions inevitably degrade them, so let me instead let her speak for herself, with the following excerpt from the prologue, which first introduced the broader mathematical community to "the liberated mathematician":

> Respected research math is dominated by men of a certain attitude. Even allowing for individual variation, there is still a tendency towards an oppressive atmosphere, which is carefully maintained and even championed by those who find it conducive to success. As any good grad student would do, I tried to fit in, mathematically. I absorbed the atmosphere and took attitudes to heart. I was miserable, and on the verge of failure. The problem was not individuals, but a system of self-preservation that, from the outside, feels like a long string of betrayals, some big, some small, perpetrated by your only

The conference that inspired these proceedings was generously funded by the National Science Foundation under grant DMS-1652600. Additional funding was provided by the Department of Mathematics at Johns Hopkins University, which hosted the Fall 2019 workshop.

> support system. When I physically removed myself from the situation, I did not know where I was or what to do. First thought: FREEDOM!!!! Second thought: but what about the others like me, who don't do math the "right way" but could still greatly contribute to the community? I combined those two thoughts and started from zero on my thesis. What resulted was a thesis written for those who do not feel that they are encouraged to be themselves. People who, for instance, try to read a math paper and think, "Oh my goodness what on earth does any of this mean why can't they just say what they mean????" rather than, "Ah, what lovely results!" (I can't even pretend to know how "normal" mathematicians feel when they read math, but I know it's not how I feel.) My thesis is, in many ways, not very serious, sometimes sarcastic, brutally honest, and very me. It is my art. It is myself. It is also as mathematically complete as I could honestly make it.
>
> I'm unwilling to pretend that all manner of ways of thinking are equally encouraged, or that there aren't very real issues of lack of diversity. It is not my place to make the system comfortable with itself. This may be challenging for happy mathematicians to read through; my only hope is that the challenge is accepted.

In her thesis [**H**] and in her blog posts [**H15**, **H16**], H points out that norms in mathematical conversation, which at its worst devotes more effort to conveying the speaker's intelligence than to effective communication, can intimidate or exclude those not versed in the art of hand-waving. Like any incisive unveiling of a fundamental truth, this seems obvious in retrospect, but until that point, I'd used linguistic tropes like "as you learned in kindergarden" or dismissive words like "obvious" and "trivial" or the subtly chilling "recall" without thinking about how these words might be perceived by students who found themselves a bit lost—how brave must someone be to admit that they don't yet understand a definition they've just been asked to recall? Since my encounter with H's writings, I've become much more attentive to the language I use in the classroom and at conferences, in one instance responding to a query about a notion I didn't have time to define with the usual dismissal—"this is a join operation"—followed by "I realize that was not an answer to that question."

This is not the first time that my world-view has been rocked by an encounter with a mathematician's writing about the profession. Early in my postdoc, I came upon Bill Thurston's essay "On proof and progress in mathematics" [**T94**], which I have since re-read many times. Thurston answers the question "what is it that mathematicians accomplish" with "what we are doing is finding ways for people to understand and think about mathematics," which certainly involves, but is not limited to, proving theorems. He then delves into the myriad challenges of communicating mathematical understanding, especially through formal channels such as papers published in journals where "writers translate their ideas into symbols and logic, and readers try to translate back." As someone who has always loved expository speaking and writing, I was inspired by Thurston's charge that

> We mathematicians need to put far greater effort into communicating mathematical ideas. To accomplish this, we need to pay much

> more attention to communicating not just our definitions, theorems, and proofs, but also our ways of thinking. We need to appreciate the value of different ways of thinking about the same mathematical structure.
>
> We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics—with consequently less energy on the most recent results. This entails developing mathematical language that is effective for the radical purpose of conveying ideas to people who don't already know them.

For someone at my career stage, I've devoted an extraordinary amount of time to writing books, notwithstanding that everyone I've asked has advised me against it. But Thurston's essay—together with a beautiful post where he responds to a query "what can one (such as myself) contribute to mathematics?" by pointing out that "mathematics only exists in a living community of mathematicians" [**T10**]—have convinced me that I can have more of an impact by doing what I love the most: sharing the mathematical ideas that I find the most beautiful.

In the solicitation for the National Science Foundation's Faculty Early Career Development Program (CAREER) grants, applicants are instructed to put forth an "integrated research and education plan" and encouraged to "think creatively about the reciprocal relationship between the proposed research and education activities and how they may inform each other in their career development as both outstanding researchers and educators" [**N20**]. To complement the more traditional aspects of my proposal, I concocted plan to host a weekend conference to provide a forum for mathematicians to critically examine the cultural practices that affect the profession. I invited mathematicians from a variety of academic institutions to present a short paper on a topic of their choosing, with ample time for audience discussion. With the aim of creating a safe space to hold an open discussion on delicate topics, I did not advertise the talks beyond my department, nor did I arrange to have them filmed, so that speakers could work though preliminary thoughts without fear that their off-the-cuff remarks will live forever on the internet.

The intention, though, was not to limit ourselves to a closed dialogue. My hope was always that the conversation on professional norms in mathematics would inspire a series of essays, like the ones that have so affected me, that could be shared with the broader mathematical community. After the workshop, the participants were invited to contribute to the conference proceedings you see here, and further invitations were extended to a few others whose experience and expertise positioned as essential contributors to this conversation.

The practice

The presenters at the September 2019 workshop delivered a diverse array of insightful talks, which set the stage for the essays included here.[1] After an inspiring, if exhausting, weekend, my co-editors Mathilde Gerbelli-Gauthier, Pamela E. Harris, Michael A. Hill, Dagan Karp, and I made plans to assemble the proceedings volume in Spring 2020.

[1] Titles and abstracts can be found on the workshop website: `http://www.math.jhu.edu/~eriehl/norms/`.

Then the world changed.

A global pandemic that was both inconceivable and recently-foreseen brought mathematics to a halt, or at least that's how it felt as my colleagues scrambled to book flights before borders closed, care for children who were suddenly confined to the home, and redesign their courses for virtual learning[2], all while trying to understand the new protocols to protect the health of their friends and family. It felt absurd to continue to daydream about abstract nonsense while so many faced existential crises.

Then the nation erupted into outraged protest in response to the latest wave of police brutality, which brought fresh attention to the systemic killings of Black Americans that had inexcusably escaped broader notice. Like many others, I participated in the Strike for Black Lives on June 10th, suggesting that the members of a working group on higher category that had planned to meet that day instead spend a few hours discussing "Diversity is a Dangerous Set-up" [**PW18**], an essay written by Chanda Prescod-Weinstein, a theoretical physicist and feminist theorist and co-organizer of the strike, that had also inspired the conversation on mathematical norms.

In parallel the President issued proclamations suspending entry of certain students and researchers from the People's Republic of China and discontinuing the H-1B and H-2B visa programs, decisions that meant that scores of our colleagues were unable to start their planned jobs. This was followed by a decision by the U.S. Immigration and Customs Enforcement to instruct the U.S. Department of State not to issue visas to students enrolled in schools or programs that are fully online (later reversed) and a new proposed rule to issue a four-year term-limit to student visas that puts our international students in an even more precarious position.

These events combined to revive the interest of our editorial team in the conversations on mathematical norms we held the past fall, and convinced us that this work to interrogate the practices of the mathematics profession is only more important now. While my co-editors and I felt a new clarity of purpose, our authors were left with an impossible task of writing something that was both timeless and timely, relevant to our present and future mathematical colleagues who might discover their words decades in the future and reflective of the unique moment we all find ourselves in today. They have more than risen to the challenge.

The Time for Miracles is Over. The volume opens with a moving personal narrative told by William Yslas Vélez and Ana Christina Vélez describing the sequence of miracles that occurred along Professor W. Y. Vélez's journey to a become a full professor at a research-intensive university. His story gives a devastating indictment of the narrow metrics used by graduate programs with the result that "yearly, mathematics selection committees cast a meager net into a sea full of dynamic and vigorous life only to haul back the same catch time after time." Meaningful traits such as creativity, perseverance, resiliency, and inner joy in learning and understanding mathematics are seldom valued as highly in students who do not fit the typical profile of the students who apply. The Vélezes adjure:

> How many more mathematically gifted people have not had the benefit of these miracles to change the direction of their lives? It should not take a miracle for a minority mathematician to obtain

[2]Virtual learning is the formal difference that arises when the participants of real synchronous learning are physically distanced.

> a position in a doctoral-granting university. The culture in mathematics departments must change if we are to take advantage of the talent that is passing through our doors.

On toxic mentorship and the academic savior complex. Pamela E. Harris follows with a taxonomy of the toxic mentors—the self-proclaimed mentor, the self-serving mentor, and the only-in-public mentor—that prey on VITAL faculty[3] and academics who remain minoritized regardless of their seniority. Some of the "academic-saviors" she has encountered:

> only wanted to make sure they could say that they were mentors to women and people of color. Being at the intersection of both, I was two birds and their "mentoring" a single stone.

At the conclusion, Harris leads us towards better mentoring and challenges us to embrace the constant improvement and lifelong learning required if we want to become good or even terrific mentors, since the impact of positive mentoring can last a lifetime.

Todxs cuentan: building community and welcoming humanity from the first day of class. Those of us who aspire to teach and practice mathematics for human flourishing [**S20**] are confronted with a challenge of how to foster a welcoming community in the classroom. Federico Ardila–Mantilla opens with an exercise in which he and his students describe something outside of mathematics that they love doing and share how they feel about being in the class because "I hoped to make clear that our full humanity was not only welcome here, but in fact would define and enrich our mathematical space."

Ardila–Mantilla emphasizes that students' sense of belonging must be nourished constantly throughout the semester. Part of this is achieved by a collaboratively developed community agreement on classroom culture and atmosphere that emphasizes the practices of *making space* and *taking space*:

> If I feel comfortable speaking out, I should be mindful of how much space I take, and make room for others. If I tend to be quieter in groups, I should remember that my ideas are important, and others will benefit from hearing them.

Ardila–Mantilla models these practices in his essay, in which he also demonstrates how to *hold space*, meaning "supporting someone by being fully present for them to process their feelings, without letting one's own feelings, ego, or proposed solutions interfere with that process" by including his students' own experiences in his classroom, told in their own words.

Congressive Question Time. Further practical advice on how to foster a supportive classroom environment, including some innovations developed in the hurried transition to online teaching, can be found in the next essay by Eugenia Cheng. In her recent book [**C20**], Cheng has proposed the terms "congressive" and "ingressive" to describe character traits characterized by "bringing things together" and "going into things," which she conceives as independent axes that can be used, for instance, to

[3] As I learned from Harris's essay, "VITAL" is an acronym coined by Rachel Levy to stand for Visitors, Instructors, TAs, Adjuncts and Lecturers.

chart an individual's responses to the COVID-19 pandemic, such as whether the decision to wear a mask is motivated by personal risk or protection of the community. These tools for analyzing societal norms are also specifically relevant to mathematicians as she observes that ingressive traits are rewarded by the mathematics profession (and society more broadly) while congressive traits tend not to be as valued by the gatekeepers at various career stages.

After she introducing these notions in her talk "Inclusion-exclusion in mathematics and beyond: who stays in, who falls out, why it happens, and what we could do about it," Cheng recalls

> it suddenly struck me that I was giving a talk in a manner completely unlike how I would ever teach any more: I was standing at the front talking, and there was going to be a raised-hands question time at the end.

She diagnoses something "ingressive" in the typical structure of post-talk question time since "Asking a question in front of a group requires someone to believe that their question is worthwhile enough to take up people's time with, and for them to have the confidence that they won't be derided or belittled for their question."

Cheng goes on to describe an experimental "congressive question time" format, and concludes with the hope that "we can learn some more congressive ways from the disasters of 2020."

Mathematics, We Have a Problem. In a challenging but vital treatise, Michelle Manes draws on her experience as an tenured faculty member, workshop organizer, and NSF program officer to talk about the myriad ways in which institutions fail to respond to sexual harassment in academia, with policies motivated more by a desire for institutional self-preservation than to provide restorative justice to victims. Manes reports that she could not have predicted how much of her time and energy would be "taken up with questions of sexual harassment by mathematicians," propelled in part by the friends, colleagues, strangers, and students who came forward with their own stories of sexual harassment with the result that "there is a fundamental disconnect between what I would choose as my service mission and what has been thrust upon me."

After her painful diagnosis of a serious problem, she proffers powerful recommended actions, from steps an individual can take to occupy a more productive space in discourse to institutional-level reforms. Manes vanquishes the straw man of false reports with the truth of ruined careers and enjoins us to "stop prioritizing men's entitlement to reputation over women's entitlement to exist without fear and humiliation." She notes that very good scientists go unfunded, and that most people who apply for funding go unfunded more often than not, so a brief pause in eligibility when repeated and plausible allegations are brought against a PI to apply should not be seen as unfairly punitive. In parting words, she puts us on notice us to demand accountability of ourselves to accept community responsiblity for reducing and preventing sexual harassment.

Fiber Bundles and Intersectional Feminism. Dagan Karp's essay begins from the premise that:

> The moduli space of genders is neither connected nor equidimensional. It is certainly not the disjoint union of two points. It is also certainly not linear with a canonical well ordering. It is not even fixed in time. So the gender binary is an incorrect model mathematically, as is a linear gender spectrum.

Instead he describes "a rich moduli space of gender identities determined by self-identification." Part of this richness derives from Kimberle Crenshaw's notion of intersectionality, deliberately framed here as "intersectional feminism." A basic premise of intersectional feminism is a mandate to acknowledge and honor intersecting aspects of identity, but on a higher level Karp notes "we must also model the intersecting nature of power structures, and the way those relate to individuals" and proposes fiber bundles as a suitable mathematical model.

The base space of the fiber bundle model of intersectional feminism is society, with an open cover consisting of "aspects of identity and their refinements." In the preimage

> lies a complex web of social structures and systems of oppression, such as racism, misogyny, homophobia and transphobia. These preimages above, the local trivializations, reveal the higher structure of intersectionality, that systems of oppression are also intersecting and cannot be separated. The transition functions preserve the structure of the bundle, illuminating the intersectional premise that the social fiber above each individual has the same structure and is preserved as we operate within biased systems.

Karp identifies the fiber with "ableist cis-heteronormative white-supremacist capitalist patriarchy" and observes that "just as points in the base space of a fiber bundle all have isomorphic fibers above, so too do all people in society live under systems of oppression." He charges us to change the fiber of our profession, for the benefit of the mathematics community and beyond.

On Parameters for Communicating Mathematics. Oliver Knill's essay lays out the myriad benefits of having faculty in a department with the time, the sensitivity, and the expertise to rise to the challenge of communicating mathematical ideas, particularly at the service course level. Knill presents useful taxonomies: "abstraction, difficulty, complexity" to label mathematical problems and "theory, example, illustration" to organize lessons.

In a section on pedagogy, Knill shares some personal observations he has made during his time in the classroom. He advocates creating a mastery learning environment for students with ample opportunity to practice basic techniques observing that "repetition is necessary in sports or music or when learning a language or programming" and "remains also a healthy habit in mathematics." Students who are able to perform algebraic computations automatically are then able to devote their precious thinking time to "make connections to other fields or reflect on whether the result 'makes sense'."

Knill's piece also includes critical reflections on the "gig workers" in mathematics departments—the VITAL faculty that Harris defends—who are hired on temporary teaching contracts, and other contingent faculty. He warns against the loss of control that results when faculty shunt administrative duties onto management professionals.

He observes that non-ladder faculty have a harder time organizing and tend to be saddled with additional work-load when graduate student workers go on strike, and can find themselves in a precarious position despite internal solidarities.

Turning Coffee into Unions: Mathematicians and Collective Bargaining. An urgent call for a fully unionized academy of "faculty, student workers, other academic staff, and of course nonacademic staff who are as much a part of the endeavor as we are" is made by Denis R. Hirschfeldt. This imperative is particularly challenging for certain mathematics faculty because "the concept of solidarity has to be learned by those with the luxury never to have had to rely on it."

Hirschfeldt acknowledges the gratitude that many of us feel that we have jobs that pay us "to do something we would want to do anyway" but argues—drawing inspiration from recent protests led by professional athletes, "who have seen the benefits of collective bargaining despite not being what one might think of as the typical kinds of workers associated with labor unions"—that we should support academic and nonacademic unions because, not despite, how fortunate we feel:

> To the extent that we feel privileged to do what we do, it is essential to understand that our work depends on a network of labor to which we do a great disservice if we stand apart from it. ...So any sense of gratitude we might have ought to be expressed by a commitment to stand in solidarity with those who make it possible directly, as well as those who have been hurt and disadvantaged by this process.

Hirschfeldt questions the long-term sustainability of the service teaching model that Knill chronicles, as free-market funding-allocation policies encourages competition between departments, and frames tenure as "increasingly becoming part of an unsustainable divide between academic haves and have-nots that threatens ultimately to swallow even the haves."

Hirschfeldt's essay, really a seminar in critical university studies, concludes with three lessons of the pandemic, the first being:

> Things are fine until they're not, $n^2 + n + 41$ is always prime until it isn't, and the cost of not being prepared can be staggering. This is true at the individual level, and all the more so at the communal one. Asking why someone without major grievances would want to unionize is like asking why a healthy person would want to get health insurance. But no, it isn't. It's like asking why a healthy person would want to have a robust public health system.

Echoing Manes' critique of Title IX policies, a second lesson is that "when things go wrong, current institutional structures will not save us." A third lesson is directed at those who recognize all the drawbacks of collective bargaining: "To riff on Churchill, collective solutions are the worst solutions to our problems except for all of those others that have been tried."

Universities in the time of climate change. The volume concludes with Izabella Łaba's prescient essay based on a presentation on the same topic that, fittingly, she elected to deliver remotely over Zoom to avoid a trip across the continent. Her

narrative interweaves apprehensions from the Fall of 2019 which "grew out of my frustration with common institutional responses to the climate emergency" with insights that take on even greater urgency in light of the global pandemic.

While administrators write "fanfics about their endless emails, memos and directives providing support and leadership during that difficult time" she warns that "this is only a small preview of what climate change will bring" and that moving all classes online for a year or two is far from the worst that can happen:

> Other pandemics may follow, as we continue to encroach on parts of the biosphere that would be best left to themselves. As wildfires ravaged Australia a few months ago, we watched helplessly the cell phone videos from residents who, having been told that it was too late to leave, sought shelter by wading into the ocean instead. Extreme conditions will become commonplace. There will be no help coming and no one available to bail us out when everyone's resources are strained to the limit.

Importantly, Łaba notes that "we should not want to return to the 'normal from February 2020. That was not sustainable even before the crisis hit." Nor is moving things "online" the solution to everything, especially when considering the energy footprint of the internet. Instead, Łaba calls for a "Green New Deal" for universities, with a redistribution of both salaries and workload, creating new jobs while allowing faculty "more time and capacity to have a life outside of work":

> Faculty numbers, especially the numbers of tenured and tenure track faculty, have no relation to how much work actually needs to be done at universities. Our workloads have long been ballooning out of control. New responsibilities are added almost every day. At the same time, faculty positions continue to be eliminated or converted to temporary ones, so that the increasing total workload is shared between fewer faculty.

Łaba contends that slower habits of work may even improve its quality. With more equality, less competition, and more cooperation—more "congressive" work environments as Cheng would describe them—mathematicians could "spend less time 'producing' new papers making incremental progress, and pay more attention to consolidation, exposition and preservation of the knowledge we already have."

Complacency is not an option, as Łaba concludes:

> Change will be forced on us. We will have to adapt, one way or another. It's up to us whether we make the transition humane and how much of human knowledge we manage to preserve. We cannot buy our way out of the climate emergency. Capitalism will not save us. Universities, as non-profit organizations dedicated to the pursuit and dissemination of knowledge, should be leading the way. We should experiment and then model the change for others.
>
> We will need to learn to make do with less. We like to say that mathematics only requires a pen and pencil. We may be tested on that.

The hope

This conversation was conceived out of a belief that when professional mathematicians have taken the time to reflect on the social forces involved in the production of mathematics, the results can be powerful. I hope you'll find these essays as provoking as I have.

Emily Riehl
Baltimore, MD
October 2020

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The Time for Miracles is Over

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In this article, "I" and "My" refer to W. Y. Vélez.

Every graduate program in the mathematical sciences undertakes the daunting task of selecting the best and brightest students based on grades in mathematics and mathematics-related subjects, courses taken, Graduate Record Exam (GRE) results and academic recommendations. The preponderance of students considered for acceptance into mathematics graduate programs have a homogenous profile—strong grades, high test-scores and exceptional recommendations. Yearly, mathematics selection committees cast a meager net into a sea full of dynamic and vigorous life only to haul back the same catch time after time. Many potentially successful students are excluded from consideration because they do not fit the typical profile of the students who apply. Mathematics graduate programs prioritize students based on their mathematical knowledge and ignore other meaningful traits such as creativity, perseverance, and resiliency. It is the exception to the rule, or dare I say miracle, that a mathematics graduate program accepts a candidate that does not fit the typical applicant profile.

Although this article deals with students at the undergraduate level, I will describe how I transitioned from an undergraduate to a graduate student and ultimately to a faculty member at a research-intensive university. It is so rare for a Chicano to accomplish this feat and even more so in the 1970s, and I would like to describe the miracles that occurred.

My family background

My pre-college education was very good. I attended Catholic schools from elementary through high school. I spent part of my sophomore year in high school at Regina Cleri Seminary where I studied to become a Catholic priest. I enjoyed the seminary life—good teachers, contemplation, lots of prayer, and three-hour study halls every evening. Although I enjoyed the experience, a life of celibacy was a price that was too high to pay.

Poverty was part of the experience for many Mexican-American households in Tucson, Arizona, and ours was no exception. We were financially poor but culturally rich. I grew up in a very Catholic household with lots of family around. Regular contact with relatives from Mexico kept us grounded to our roots, and I grew up with a sense of pride for my culture and speaking Spanish. My deep connections to Mexican culture, music, and language have anchored me throughout my life.

When I was five-years-old, we almost lost our home. In order to remain in our home, my parents rented it out but kept two rooms. My parents, sister and two brothers slept in one room and a small trailer. The other room served as a kitchen. We had no bathroom or running water in either of the two rooms. My father was a mechanic and his garage was next door to us. Fortunately, my father sold the garage to a friend and we used the bathroom and shower there. When I was nine my father died, adding grief to our challenges. By the time I was a teenager, we were able to move into the entire house. My mother was determined to keep a roof over our heads and she devoted all of her energy to that endeavor.

I was a "B" student in high school and had no idea what I was going to study in college. I was fortunate that the University of Arizona (UA) was in Tucson. It never occurred to me to apply to any other university since I barely had enough money for in-state tuition. My brothers and I all attended the UA, and lived at home until we married.

In my senior year of high school, I attended a talk given by a chemical engineer. He reported his salary to be $11,000 per year, an unheard of amount of money. My mother worked three jobs and probably never earned more than $3,000 in a year. Because of that talk, I enrolled in chemical engineering at the UA in 1964. I was well prepared academically, as I took mathematics and science courses each year in high school. Yet my first semester at the UA was a disaster.

My undergraduate career path

During my undergraduate years, the history of mathematics and the lives of mathematicians fascinated me. I read many biographies about the most influential and universal mathematicians of the 19th and early 20th centuries. One anecdote from a biography stands out in my memory because it illustrates the insular and pedagogical nature of many leading mathematicians from the past. Upon learning that one of his students had married, the mathematician dispatched him from academics immediately. This mathematician believed that one could only be truly dedicated to mathematics by forsaking all other relationships and interests. Though I would like to believe that such attitudes have disappeared, Denis R. Hirschfeldt (see this volume) in arguing for the unionization of faculty and graduate students, states that this romantic view of our profession is one that still calls for total commitment.

Restrictive attitudes have dissuaded students from pursuing careers in the mathematical sciences. It is only by a series of miracles that I earned a Ph.D. in mathematics and obtained a position in a doctoral-granting university. Out of my cohort of 25 students who started the graduate program in 1970, I was the only one to complete a Ph. D. at the UA. I was the first Chicano to obtain a doctoral degree in mathematics at the University of Arizona (UA) in 1975. In 1977, I accepted a position as an Assistant

Professor in the mathematics department at the UA. I remained the only Chicano in a tenure-track position for the next 41 years until my retirement in 2018.

As a first generation college student, I desperately needed solid advising and mentoring. The importance of advising and mentoring on a college campus is currently widely accepted as best practice but this was not the case in 1964 when I was a freshman at the UA. The only kind of advice that I received was bad advice. One mathematics faculty member suggested that I change my major to Spanish as he surmised that this was the only feasible course of study for an individual with my background. During my senior year, I was elated when a visiting faculty member commented that I exhibited some mathematical talent and suggested that I take his reading course in group theory. When I sought permission from the department the requisite faculty member took one look at my transcripts, denied my request and informed me, "Reading courses are only for the better students".

Pamela Harris (see this volume) describes some of the "toxic mentors" that appeared along her career path. It is a sad fact that many of us can recount similar experiences. I am reminded of the Mexican saying, "Lo que no mata, engorda" (If it does not kill you, it will make you stronger). This applies to all of us who have survived these indignities. This makes me wonder how many potential mathematicians had their interest in mathematics killed because they did not have the temperament to survive this kind of treatment. Our survival, and these losses, have shaped the character of our profession. Fortunately, Pamela Harris' article presents a more positive agenda for mentors.

In my first semester I enrolled in calculus, dropped it after three weeks, and replaced it with college algebra and trigonometry. I earned nine units of Ds. The only "A" that I earned was in weight training. How could I have performed so poorly? I took college algebra and trigonometry in high school and had good study habits. Yet I almost failed several courses. If I could not pass material that I learned in high school, how would I be successful in advancing to more complex mathematical subjects such as calculus? My pride had suffered a calamitous blow. I immediately dropped engineering as a major.

COURSE DESCRIPTION	DEPARTMENT	COURSE NO.	UNITS	GRADE
1ST SEM 1964-65				
ANALY GEOM & CALC	MATH	79A	5	8
FRESHMN COMPOSITION	ENGL	1	3	3
WEIGHT TRAINING	HPER	11	1	1
ENGR GRAPHICS	ENGR	10	3	3
FIRST YEAR BASIC	ML S	1A	1	3
FUND OF CHEMISTRY	CHEM	2A	4	4
COLL ALGEBRA & TRIG	MATH	44	5	4
		TOTAL	17	

FIGURE 1. First Semester.

I will present my undergraduate transcripts and ask you to review it as though you were my undergraduate advisor. Many of you would have to agree with the assessment

that I was not "one of the better students", as my grades were miserable compared to the typical mathematics major heading towards graduate school. What chance do you think I have of acceptance into a mathematics graduate program? How likely is it that I would complete graduate school and earn a Ph.D. in mathematics? What should my next step be?

After surveying the shoddy results of my first semester in college, I felt despondent and unsure of my academic future. As I walked over to the Student Union one day, a Navy recruiter called me over to his table. He told me that if I could survive for two years in college I could become a pilot in the U.S. Navy. Although I did not aspire to be a navy pilot, my academic future looked rather dim at the time so I joined the U.S. Navy Reserves. In the summer of 1965 I went to boot camp in San Diego, California. The reserve commitment required me to report for duty one weekend a month and two weeks in the summer on a navy ship for the next three years.

I was determined to take calculus again and complete it successfully. I learned an important lesson the semester before and took a lighter load in an effort to increase my grade point average. Without a declared major or plan of study, my future was a complete mystery. Nobody in my family could provide academic advice. Nor could I turn to any kind of advising program to assist me in investigating possible career options. Aside from calculus, the other courses just satisfied university requirements. My academic career path felt untethered and unfocused. Many first-generation college students face the same uncharted territory when arriving at a university campus. Moreover, the mathematics classroom can be an uninviting space, especially for students whose culture is not represented among the teaching staff. Federico Ardila (see this volume) provides an example of an activity that he created for his classroom that served to not only welcome students but also to validate their culture in a university setting.

2ND SEM 1964-65				
ELEM PSYCHOLOGY	PSYC	1A	3	2
ANALY GEOM & CALC	MATH	79A	5	3
FRESHMN COMPOSITION	ENGL	3	3	3
HANDBALL	HPER	6	1	1
ELEMENTARY SPANISH	SPAN	1A	4	1
		TOTAL	16	

FIGURE 2. Second Semester.

As I began my second year, I still felt completely lost. As I mentioned before I do not remember any faculty advising. Fellow undergraduates and some of the graduate students that I met served as my advisors. Contrary to logic and good sense, I found myself signing up for calculus. Again. As I reflect on that decision, I believe that good old-fashioned stubbornness motivated me. Calculus had injured my pride and I was not going to let it take me down. I think my religious upbringing also swayed me towards mathematics. Mathematics is like theology but with nicer logical paths to follow. I had not had physics in high school so I decided to take it in my third semester.

In my third semester, I made a very odd decision. I decided I was going to earn a Ph.D. in mathematics! My grades were mediocre, yet I was fascinated by my readings

COURSE	DESCRIPTION	DEPARTMENT	COURSE NO.	HONORS SECTION	UNITS	GRADE
142448	VELEZ WILLIAM	YSLAS	1ST SEM	65-66		
ANALY	GEOM + CALC	MATH	79B		5	1
INTRO	TO PHYSICS	PHYS	2A		4	2
INTRO	TO SOCIOLOGY	SOC	1A		3	8
INTERMED	SPANISH	SPAN	3B		4	1
				TOTAL	13	

FIGURE 3. Third Semester.

COURSE	DESCRIPTION	DEPARTMENT	COURSE NO.	HONORS SECTION	UNITS	GRADE
142448	VELEZ WILLIAM	YSLAS	2ND SEM	65-66		
CALCULUS		MATH	80		2	3
INT ANAL	DIFF EQUAT	MATH	181		3	2
INTRO TO	PHYSICS	PHYS	2B		4	2
FOUNDATION	SCI PHYS	PHYS	12		3	1
ELEM	PSYCHOLOGY	PSYC	1B		3	2
INTER	COMP + CONVER	SPAN	75A		2	1
				TOTAL	17	

FIGURE 4. Fourth Semester.

in the history of mathematics, the many conversations I had with my mathematical friends, and the mathematical connections that I began to see. However, the highlight of the year was meeting my future wife, Bernice Lopez. I fell madly in love. I had no job, no money, no car and a used a bicycle to get around. Many years later one of Bernice's cousins told me how annoyed her father became when I would show up at their home on my second-hand bicycle, greet Bernice, and get in her new car to go on a date. Efren, Bernice's father, would see me coming through the kitchen window and say, "Allí viene ese pinchi pelado en su bicicleta!" (Here comes that broke SOB on his bicycle). Efren's wife, Maria, would do her best to soothe him saying, "Efren, no digas nada." (Efren, do not say a word). Poor man. I was not the husband he envisioned for his beautiful first-born daughter.

Notice I earned an A in complex variables. The professor in that course gave everyone "As". I began seriously considering my graduate school options. Bernice and I made plans for a June 1968 wedding, followed by a honeymoon at the Grand Canyon. In my junior year I got a job at the U.S. post office, working 20 hours per week. I earned enough money to buy an engagement ring and save for our wedding expenses. We had to pay for our wedding since our parents were not financially able to take on this expense.

In my seventh semester, I took analysis and we used R. Creighton Buck's book. I was in heaven and earned an "honest A" in the course. I completed all of the problems assigned and sometimes provided two proofs, which drove the instructor to insanity. This course confirmed that I belonged in mathematics. My grades finally matched my interest and determination to study abstract mathematics. I completed the requirements for the mathematics major and physics minor that seventh semester, and only 4 more units were needed to complete a Bachelor of Science degree. I enrolled in a

142448	VELEZ WILLIAM	YSLAS	1ST SEM	66-67		
VOICE CLASS		MUS	26A		1	7
FOUND OF GEOMETRY		MATH	213A		3	2
INTRO MOD ALGEBRA		MATH	231		3	3
HISTORY OF MATH		MATH	240		3	2
INTRO TO LOGIC		PHIL	12		3	1
ADV GENERAL PHYSICS		PHYS	101A		2	2
				TOTAL	14	
142448	VELEZ WILLIAM	YSLAS	2ND SEM	66-67		
STUDY OF ENGL WORDS		CLAS	15B		2	1
ELEM COMPLEX VARBLS		MATH	203		3	1
LINEAR ALGEBRA		MATH	232		3	2
INTRO SYMBOL LOGIC		PHIL	25		3	2
ADV GENERAL PHYSICS		PHYS	101B		2	1
STRUCTURE OF MATTER		PHYS	130		3	2
				TOTAL	16	

FIGURE 5. Fifth & Sixth Semesters.

graduate-level course and thought it best to take a lighter load, so I only enrolled in 10 units. In preparation for the graduate school application process, I took the Graduate Record Exam (GRE) subject test with disappointing results. If I had a copy of my GRE score, I would include it here. I applied to the University of California Los Angeles (UCLA) and the University of Illinois Urbana Champaign (UIUC) graduate schools, and unsurprisingly they both rejected me.

NAME - VELEZ, WILLIAM YSLAS MATRIC. NO.

COURSE	DESCRIPTION	DEPARTMENT	COURSE NO	HONORS SECTION	UNITS	GRADE
142448	VELEZ WILLIAM	YSLAS	SUM SEM	1967		
2ND TERM						
INTRC TO HUMANITIES		HUM	50B		4	1
				TOTAL	04	
CREDIT FOR SERVICE IN THE U.S. NAVAL RESERVE ACTIVE						
BASIC MIL. SCI.		M.S.			3	
					3	
142448	VELEZ WILLIAM	YSLAS	1ST SEM	67/68		
INTRC TO PHILOSOPHY		PHIL	11		3	8
THEOR MECHANICS 1		PHYS	210		3	2
INTRC TO ANALYSIS		MATH	280A		3	1
THEORY PROBABILITY		MATH	290A		3	3
MATHEMATICAL LOGIC		MATH	295A		3	1
				TOTAL	12	

FIGURE 6. Last Semester.

I received a letter in February of 1968 ordering me to active duty on March 18. If you recall I enlisted in the U.S. Naval Reserves during my first year at the university that included two years of active service in the U.S. Navy. Regrettably, I failed to understand the requirements of the active duty deferral. The military stipulated that a deferral from active duty was for full-time students only. Full-time meant 12 units. I was taking 10. Guess who was heading off to war? I learned that if the Navy postponed my report date by one week, the university would give me half credit for my courses. After many

phone calls, I made this change. I reported for active duty having completed my degree in less than four years.

You can imagine the panic that my active duty status brought on. The print on the wedding invitations was still wet and many of the arrangements were set. And of course there was the small detail of my graduation from the university. Bernice was in a panic and my mother in tears to think of one of her sons going to war. This was the Vietnam era.

On March 28, I reported to the U.S. Naval Station in Long Beach, California for active duty. To add to the pandemonium my mother called me during the first week of April and told me that a letter from Purdue University had arrived. To my great pleasure on April 1, 1968 Purdue University accepted me as a graduate student and offered me a teaching assistantship. I was going to study mathematics at Purdue University and become a mathematician! Unbeknownst to me a big surprise lay in store for me that was to derail my highly anticipated Purdue dreams.

I wrote to Purdue University, informed them that the military summoned me to active duty, and requested that they postpone their offer until I returned. I received a perfunctory letter from Purdue University stating that they were "under no contractual agreement with me" since I had been activated before the letter arrived. As was so common during the Vietnam War era there was no "thank you for your service". By April, I was in the Tonkin Gulf on the aircraft carrier, U.S.S. Yorktown, and assigned dishwashing duty for the first several weeks.

Reflections on my career path

Most faculty assume that earning a "C" in a mathematics course means that a student does not have what it takes to study advanced mathematics. I earned plenty of "Cs" but continued because understanding the structure in mathematics fascinated me. Most mathematics faculty members sail through their undergraduate years, assume that students with less than a stellar undergraduate performance could not possibly have mathematical talent, and dismiss them without consideration. Undergraduate students who express an interest in mathematics should be encouraged to pursue their mathematical interests in spite of being average.

The purpose of advising is to listen to the advisee and suggest a path that leads them to their own goal.

I would like to review some of my experiences as an undergraduate, not to complain but to point out the attitudes that mathematicians have about students can adversely impact their decisions to continue their studies in mathematics.

1. How does one advise a student like myself? It is a real challenge to provide good advice. My attitude towards students has always been that if a student wants to continue in the study of mathematics then I am there to support that student. My function as an advisor is to present a mathematical path to help students reach their goals. I should point out that I have helped students with poorer academic records than mine get into graduate school in the mathematical sciences.
2. Entering students do not get good advice and often take courses that merely fulfill requirements, rather than courses that explore possible careers or areas of study.

3. In my second semester I retook calculus. I recall the professor directed his comments to one individual student who sat in the front row. It was almost as if they were having a personal conversation. The rest of us felt ignored. We were not smart enough to be part of the class.
4. My biggest complaint about the faculty that I had in undergraduate courses was that they looked bored and lacked enthusiasm for their subject. I always wondered how they mustered up enough energy to obtain their doctoral degrees?
5. Following up on this, there is a lack of emotion in mathematics classes. Mathematical instruction tends to be very dry, as if rational thought replaces the emotional connection we have to our subject.
6. There is so much going on in a student's life. Sometimes life impinges on a student's ability to concentrate on their studies. The cultural background of many students does not often inform them of the opportunities available to mathematically trained students.
7. I took advanced calculus in my senior year and this class was critical to me in solidifying my intent to pursue graduate studies. The fact that I took this course in my senior year is fairly common for mathematics majors in the U.S. If you talk to your international colleagues, many will tell you that this was the first course they took in college. What programs did international faculty have in their country of origin to help them develop mathematical maturity so early in their careers? Most U.S. students do not have access to these programs. Fortunately, the U.S. educational system provides an opportunity for high school students to mature in college enough to be able to handle the abstract concepts and move on to pursue graduate studies. It is important for graduate programs to understand our own educational system.

As I have come to the end of my career, I have done some thinking as to why my grades were so poor. I have worked for ACT (American College Testing) for more than twenty years, reviewing their problem sets. It is embarrassing the mistakes that I make in going over their problems. When I was doing a very simple algebra problem recently I identified a behavior of mine that I think was behind these poor grades.

Problem: A rectangle has a length that is 4 times its width and a perimeter that is 50 ft. What are the dimensions of that rectangle?

I drew a rectangle and denoted its sides by w and 5w. Then I wrote the equation $6w = 25$.

I am in a hurry to move on. That has always been my problem. When I read the problem, I immediately calculated half of the perimeter to be 5w and then carelessly denoted the length of the rectangle with that quantity. I commit these types of errors repeatedly. I turned out to be a good graduate student. In graduate school, we need to provide careful proofs, not be good accountants.

The time for miracles is over

I retired as a full professor, and had a spectacular academic life. I have spoken to many mathematicians over the years who describe how difficult graduate school was, and how they almost quit at some point.

My career path was much different. I failed as a first year undergraduate student and barely survived. By my third semester, when I was 18, I decided to earn a Ph.D. in mathematics. After that point, my entire academic life was a success and a joy. My grades did not reflect academic success, but that metric could not measure my inner joy in learning and understanding mathematics. I never doubted my decision. I found a wonderful partner in life and was totally engaged in learning mathematics, creating and applying mathematics, teaching wonderful ideas, and in the latter part of my life, convincing students that mathematics should be part of their lives.

And this almost did not happen!

When I returned from Vietnam, I again applied to several graduate programs. As expected, all of the out-of-state schools turned me down, though I expected that Purdue would be there for me. I was surprised when Purdue this time turned me down. I guess that they looked at my grades in my last semester, saw that I earned a "C" in a graduate course in my last semester, and decided that I was not worth supporting. That last semester lasted only eight weeks for me and the last four weeks were hectic and emotional. I was going off to war, my wedding plans were canceled, my family was in turmoil. This is an example of the kind of tragedy that students have to deal with. No graduate program corresponded with me to ask about my circumstances.

It was now April 1970 and I had several rejection letters. Then a very curious thing happened. I describe the following conversation between a man calling the license bureau in Tucson and the woman answering the phone.

Man: "There is a car blocking my driveway. Here is the license number of the car. Can I have the name and phone number of the owner of the car?"

Woman: "I can't give you that information. I have no idea who you are or if you are trustworthy."

Man: "Oh, I'm very trustworthy. I am Dr. Jim Clay, head of the mathematics department at the University."

Woman: "And why have you not offered my son a teaching assistantship?"

The next week I had an offer of a teaching assistantship. No, Dr. Clay did not get that information.

I came so close to not having the opportunity to earn a doctorate. I call this a miracle. My entering class of graduate students in 1970 was about 25 students. I was the only one from that group to earn a Ph.D. in mathematics from the University of Arizona.

After completing my doctoral degree in 1975, I did not want an academic position. Instead, I accepted a position as a Member of the Technical Staff at Sandia Laboratories in New Mexico right after graduate school. However late in the fall semester of 1975 Sandia Laboratories contacted me and informed me that there was a hiring freeze. I spoke to my thesis advisor, Dr. Henry B. Mann, told him of my predicament, and he suggested that I speak with the department head, Dr. Hanno Rund to seek his advice and inquire about any kind of temporary position. Dr. Rund said that his funds were committed. However, the next day, Dr. Rund came to see me. In my office! He entered my office, closed the door behind him, and said the most amazing thing had happened. Dr. Mann told Dr. Rund that if he offered me an Assistant Professorship, he would retire early. Dr. Rund offered me the position of Assistant Professor. I politely turned him down, telling him that I was not interested in being a professor. The hiring freeze lifted

and I went to Sandia Laboratories. Two years later, I missed teaching and contacted the University of Arizona to see if there was the possibility of employment. Dr. Rund asked me to return to the UA and give a talk. He made me an offer and the rest is history. I retired from the University of Arizona in 2018 after 41 years.

Had it not been for Henry Mann I would not have had such a wonderful position.

Over the years, I have heard people say the reason the university hired me was due to my status as a minority. But, I think it is was due to a miracle. How many more mathematically gifted people have not had the benefit of these miracles to change the direction of their lives? It should not take a miracle for a minority mathematician to obtain a position in a doctoral-granting university. The culture in mathematics departments must change if we are to take advantage of the talent that is passing through our doors.

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On toxic mentorship and the academic savior complex

Pamela E. Harris

To my students—may your mentoring experiences be positive

ABSTRACT. Even with the best intentions, faculty can fall prey to the academic savior complex, where they aim to "save" faculty from professional failure. Such faculty members may believe that young faculty are incapable of determining what is important in their academic careers, try to convince others what to think or do, offer advice and direction without being asked, and seek to feel needed in order to have a positive relationship with others. This behavior is easily masked as mentoring and creates very toxic environments for underrepresented faculty. In this article, I share some of my experiences working with faculty whose academic savior complex was illustrated through toxic mentoring that negatively affected my career progression and I contrast it with ways in which good/positive mentoring could have taken place.

Upon reading this article's title you may ask yourself: *What is the academic savior complex?* As is customary in mathematics, let me begin this article with a definition.

DEFINITION 0.1. The *X savior complex* refers to a person who identifies as *X* and who acts or feigns to help non-*X* people, with this help being self-serving or self-beneficial.

In this article, I let *X* be a **academic** and non-*X* is a **non-academic**. More precisely, I will narrow the story/study by referring to an academic as someone who already identifies as part of the academy. More precisely, a senior tenured professor, often from privileged backgrounds. In this case, non-*X* includes VITAL faculty[1] and those minoritized in academics, who are minoritized regardless of their seniority. Throughout, I use the phrase "toxic mentoring" to refer to the experience of a mentee upon being on the receiving end of a mentor suffering from the academic savior complex.

I organize this article as follows. In Section 1 I describe my personal journey through mathematics. In Section 2 I present three distinct types of toxic mentoring experienced through this journey, and I contrast it with what would have made those experiences more positive. I end with Section 3 where I provide some of the ways I continue to grow and develop to become a better mentor.

Acknowledgments. My unending thanks go out to my family who have helped me process the trauma of terrible mentoring, and to all of the good mentors who have shown me the difference. I would also like to thank Michelle Manes, Luis Leyva, Adriana Salerno, and Francis Su who provided resources shared in Section 3.

[1]VITAL stands for Visitors, Instructors, TAs, Adjuncts and Lecturers [**8**].

1. How did I get here anyways?

I am a cisgender woman of color. I was an undocumented and first-generation college student having immigrated from Mexico to the United States at the age of 12. I began my higher education journey by attending community college. After completing two associate degrees, I transferred to a four-year Jesuit college to complete my Bachelor's degree. After a semester off for the birth of my daughter Akira, I began graduate school at a large public graduate school where I completed an MS and a PhD in mathematics. Upon graduation, I became a postdoctoral fellow working at a military academy and I am now a tenured associate professor at a small liberal arts college in New England.

By every reasonable measurement of academic progress, I have succeeded. I credit this success in large part to the good mentors I had throughout my career. Luckily, these mentors' belief in my potential far exceeded my own and they were instrumental in the life choices I made. Given my inexperience with navigating an academic career, I often found myself completely unaware of what I should do, what opportunities existed, and which I should pursue/avoid to develop as an academic. This means that at times I followed their advice blindly and without question. After all, what could possibly go wrong?

Well some things definitely went wrong, and these are the stories on which I will focus. Although, it is always much more pleasant to recount when things turn out wonderfully, and it is much harder to put in writing those things that caused pain and damage. However, in order to grow we must learn from those mistakes, which is the motivation for putting these stories on paper. My hope is that these stories leave a mark on those who read them—a mark that helps them be better mentors, in particular to their underrepresented students.

1.1. What is a mentor? This is the first central question we should consider. According to Merriam-Webster, a mentor is a trusted counselor or guide [**3**]. The underlying assumption is that a mentor is by default a "good" mentor and their mentoring leads to a positive relationship and positive outcomes. Yet, people are so multidimensional and imperfect and in constant change, that one cannot expect that a good mentor will for ever remain a good mentor. Also, what good/positive mentoring means for someone may be completely abusive for someone else.

My lived experiences have taught me that what we can do is set certain expectations/roles of a good/positive mentoring relationship. Such relationships should, at a minimum demand that those involved value each other as people, develop mutual trust and respect, and listen both to what is being said and how it is being said. In addition, the mentor must aim to help the mentee solve their own problems, rather than give direction, and the mentor should focus on the mentee's development while resisting the urge to produce a clone.

This last piece is rather difficult to do in an academic setting, where a PhD advisor is often developing their mini-me, and where their success as an advisor is highly tied to their student's success. However, a PhD advisor does not need to fulfill every possible mentoring role—nor can one expect this of any single mentor. I do think it is very difficult for those who gain from someone's success to be a good mentor. Such a setting simply muddies the waters and adds additional pressure to the relationship. Making

it difficult to separate what is actual mentoring versus self-serving advice. In my professional journey, the mentors who continue to have the most positive impact in my career have never been my professors/advisors/supervisors. They have gained nothing from my professional advancement—aside from the associated happiness from seeing me succeed. Yet toxic mentoring is real and has deeply negative effects. Let me discuss three such experiences.

2. Examples of toxic mentoring

I have been the recipient of some rather awful and toxic mentoring. Although I acknowledge that good intentions may have been driving the mentoring, the actual mentoring caused a lot of damage. I focus on three types of toxic mentors: the self-proclaimed mentor, the self-serving mentor, and the only-in-public mentor.[2]

2.1. The self-proclaimed mentor. My self-proclaimed mentor was someone who volunteered to be my mentor and who promptly began listing me in their CV as their mentee. Their mentoring was border-line abusive. They only wanted to make sure they could say that they were mentors to women and people of color. Being at the intersection of both, I was two birds and their mentoring a single stone.

I spent two years "mentored" by this person, only to find myself constantly depressed and full of self-doubt. Their advice always went against what I had heard or believed to be true. Yet, this person was a well-respected scholar (or so I thought) and I was a recent PhD eager to work hard and become a better mathematician. Like in an abusive relationship, there were many highs and lows. Through this time, my research began to blossom, and I vividly recall having a paper published in a journal and sharing the good news. They said *"You need to publish in better journals. Do you even know where you should be submitting? This journal is level C. It will not count for much when you go up for tenure."* What could have been a simple lesson about the quality of academic journals and how to determine such things, their "mentoring" disillusioned me regarding the publishing process.[3] This experience left me feeling numb and made me more unsure of my abilities to succeed as an academic.

In contrast, a good mentor would have congratulated me about the publication and found a different time and way to share information about the norms of publishing mathematical papers. A good mentor would have offered to look at my work prior to my submission. A good mentor would have taken the time to show me how to find/determine journal rankings and advised me on ways one traverses the mathematical publishing world. From advising on potential journals, how to reach out to editors, how to respond to referee reports, including information on rejections and acceptances, this mentor fell very short on building a positive mentoring relationship.

However, this "mentor" continued to list me as a mentee, even with that C level publication. I later discovered that they were getting ready to submit a packet to go

[2]An initial detail of these mentoring experiences was originally published on the AMS eMentoring Network blog [**2**].

[3]For the record, I still think that particular paper was actually quite good, regardless of the ranking of the journal. In fact years after that particular publication, this journal has many papers from leaders in the field. So even if the rankings of that particular journal were not the best at the time I published there, we must remember that journal rankings change over time.

up for full professor at their institution and mentoring was large component of that evaluation. Whether that mentoring was positive was not necessary.

2.2. The self-serving mentor. Their advice was based only on what they thought they could gain from me and my work. They asked me to work at all hours of the night. It even once led to a call one late evening, when a preprint I sent them did not contain their name as an author. You see, they suggested the problem, but did not contribute to the solution nor the writing. In contrast to their elite ivy league education, they said that my low-quality graduate education never taught me what merits coauthor-ship. They also assured me that they would write letters of support for me saying I did most of the work.

They asked me to work at a pace that was unhealthy to me. They said that publishing at a fast rate is the only way that I would be a successful academic. Any push back led to their explanation that completing the research was more important for me than for any of our other collaborators. After all, I was the one starting a tenure track position at an elite liberal arts institution.

They also berated and disparaged my outreach and service work. This was work that made me feel connected to the scientific community, allowed me to meet and collaborate with other underrepresented scientists, and which brought me great joy. None of that mattered. They only saw that work as taking time away from the research they wanted me to complete. This was not mentoring since every piece of advice was self-serving, as they would benefit from any work I did.

In contrast, a good mentor would give open problems freely. A good mentor would not expect being listed as a coauthor on a manuscript when they did not contribute to the research. A good mentor would remark that a thanks in an acknowledgement section would be more than sufficient. I understand that some may disagree with this position, but in combinatorics (my research area), it is rather common for mathematicians to share open problems freely with more junior faculty. Good mentors even hold entire conferences focused on open problems and on building actual collaborations [**1**, **5**, **6**]. Whether or not such conferences exist within other mathematical areas, we ought to agree that the self-serving mentor is less mentor and more leech. Identifying them early and avoiding them quickly is the only option for the well-being of junior faculty.

2.3. The only-in-public mentor. This type of toxic mentoring has been the most emotionally challenging. In a very public way, this person plays the role of a big supporter of my work, while in private they provide me with very toxic advice. I will make this concrete with a very specific example.

I once posted on social media a call to other academics to also request service work from white cisgender men. This was in light of seeing many other women of color being completely overwhelmed with the level of service invitations they were receiving. These were requests to do things as mundane as fold t-shirts for an event, or bake cookies for faculty meetings, or to clean up communal areas in their department after events/meetings. This mentor did not know all of the details, and their response to me in private asked me to not share such thoughts in public. They reminded me that they had worked for years to have white colleagues extend invitations to women of color. I pointed out that what I was speaking of was not about speaking invitations nor research

collaborations. His words still ring in my ears (and my inbox): *"... if you're getting an invitation for service this means they want to start a dialogue. If women of color don't make this sacrifice then the dialogues don't happen. Someone has to make the sacrifice. It's sad but I don't know who else can."*

This "mentor" who praises me in public is now asking me to continue to sacrifice more time to service commitments. They are convinced this would start conversations that could lead to some potential research collaborations. Years of evidence has shown me this is extremely unlikely to occur.

In contrast a good mentor would have listened more and given their opinion less. A good mentor would not ask me to silence my voice nor my thoughts. A good mentor would have helped me find concrete ways in which I could have turned service requests into research or leadership opportunities. Never, would a good mentor ask those from the most marginalized groups to continue to sacrifice for some future theoretical benefit. A good mentor would have put me and my feelings first.

3. Toward better mentoring

The stories above are obvious examples of toxic mentoring. Through those experiences, I have come to the realization there is no one fits-all universal mentor. Instead, I have learned to not be discouraged when a mentor fails me. It is not my fault, and often it is also not their fault. After all, as mathematicians we are not trained as mentors. Also, some mentors may only be for a season, while others can be for a lifetime. By reflecting on my goals, I have been better able to determine which mentors are which.

Moreover, I have also learned to be very specific on what I need as a mentee. This small but direct change in my behavior has vastly reduced the amount of toxic mentoring I have experienced. I highly suggest that everyone create their own network of mentors and analyze it through a Mentoring Map [**4**]. Such a plan has allowed me to determine my unmet mentoring needs. From which I can then verbalize what it is that I am needing from a mentor.

By being more involved with organizations which emphasize good mentoring, I have also grown tremendously as a mentee and mentor. This includes my involvement with the Association for Women in Mathematics, the Society for the Advancement of Chicanos/Hispanics and Native Americans in Science (SACNAS), and the Math Alliance. For example, SACNAS provides a variety of ways to be involved in mentoring a new generation for scientists while at their annual conference. You can become a poster judge, where you are explicitly instructed to "provide positive and constructive feedback to presenters" and you can be involved in the Conversations with a Scientists, where you participate in small round table discussions about your educational and professional journey as a scientist. Participating in these events has given me the experience of receiving positive mentoring from others and has helped me develop a stronger sense of community within the mathematical sciences. This latter has helped me become a better mentor, as I now have experienced what it feels like to be fully myself as a scientist.

I am also a strong supporter of lateral mentoring relationships. This is mentoring between people at the same academic stages. It should be of no surprise that mentoring does not always have to be vertical. In fact, lateral mentoring has made me more acutely aware of the commonness of some of the challenges I have faced at certain

critical stages within my academic career. These lateral mentoring relationships have given me a sense of purpose to advance in my career and to pursue further leadership opportunities. My motivation is to help remedy some of these challenges and to be better prepared to mentor those who are experiencing them.

Within the mathematical sciences, we have (thankfully) begun to place much more of an emphasis on good mentoring. Even mathematicians known primarily for their research contributions acknowledge that there is room for growth in this direction. Leading by example, Bernd Sturmfels described his own experiences as a mentor and the changes he has made in order to develop as a good mentor. He explains that "[t]here are many ways... to be a terrific mentor" and gives concrete advice on how he has improved as a mentor [**7**]. Such pieces provide a few of the many resources available to those who are interested in becoming better mentors.

For me being a good mentor is intrinsically tied to being a good human being: A good mentor must value the mentee as a person. Of course this requires there to be a solid relationship that is built on trust and, more importantly, respect. A good mentor offers advice, but the advice will be directly related to how the mentee may be able to solve a particular problem/challenge that they are experiencing. That is, a good mentor provided possible ideas, and not directions, and is never upset if the mentee does not take their advice. I think of a good mentor as a solid sounding board: someone who listens intently and provides a multitude of potential solutions—rather than telling me what they would do. Most importantly, and part of what is most challenging in academic settings, is that a good mentor has to have a mentee's development as their primary objective, and should at all costs avoid trying to force their mentee to become their mini-me.

Being a good mentor requires a true commitment to helping a mentee reach their personal and professional goals. Being a good mentor is not easy and none of us are taught how to be a good mentor. This does not mean that we should not try to be better. After all, constant improvement and lifelong learning are a part of what drew us to an academic profession, is it not? If that is the case, then we must continue to do more to better ourselves as mentors. The impact of positive mentoring will be long lasting and will improve our community at large. So get out there and become a better mentor. We owe it to our students and peers to become the mentors they need and deserve.

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Todxs cuentan: building community and welcoming humanity from the first day of class.

Federico Ardila–Mantilla

ABSTRACT. Everyone can have joyful, meaningful, and empowering academic experiences; but no single academic experience is joyful, meaningful, and empowering to everyone. Is it possible to build academic spaces where every participant can thrive? How might we do that? Audre Lorde advises us to use our differences to our advantage. bell hooks highlights the key role of building community while addressing power dynamics. Rochelle Gutiérrez emphasizes the importance of welcoming students' full humanity. This note discusses some efforts to implement these ideas in a university classroom, focusing the discussion on the first few days of class.

4. Community.

> Excitement about ideas [is] not sufficient to create an exciting learning process. As a classroom community, our capacity to generate excitement is deeply affected by our interest in one another, in hearing one another's voices, in recognizing one another's presence. Any radical pedagogy must insist that everyone's presence is acknowledged. That insistence cannot be simply stated. It has to be demonstrated through pedagogical practices. There must be an ongoing recognition that everyone influences the classroom dynamic, that everyone contributes. Often before this process can begin there has to be some deconstruction of the traditional notion that only the professor is responsible for classroom dynamics.
>
> bell hooks [**bh**]

5. January, 2017: The week before class.

The week before the semester started, as usual, I found myself frantically trying to organize my office, our apartment, our record collection, anything else that needed or did not need organizing. This made me feel productive while I avoided preparing for my upcoming classes.

While I was organizing our living room, I found a portable turntable and far too many records that did not fit in our crates. I brought them to my office the next day. When I played the first record, I was pleasantly surprised by how drastically this addition transformed the space—just like the extra coffee maker I had brought a few

semesters ago, which allowed me to offer visitors a nice, strong cafecito before we began to talk about life, or mathematics, or both.

That semester, I was going to teach my combinatorics class in a dark room with small windows and broken blinds. The whiteboards on all the walls would be very useful for group work; but the long rows of tables nailed to the ground and the clunky laptop computer locked into place on every seat would make collaboration challenging. A few hours before class, I was still thinking about how to make students feel welcome in this space.

I knew that the first days of class would heavily influence how the classroom would feel throughout the semester. This class had a very broad range of students: from second-year undergraduates to Master's students doing research in the field. Most of them did not know each other and I was dreading the uncomfortable silence that can sometimes engulf the room before class starts. So I thought: "I should at least bring my turntable and a few records to class."

When students arrived on the first day of class, Carlos Embales was playing. This quickly broke the ice and seemed to give them permission to start talking to each other.

6. Introductions.

We got in a circle—or as close to a circle as the tables allowed—and answered a few questions:

- What would you like us to call you?
- What is something outside of mathematics that you love doing?
- How do you feel about being here?

I answered first, to give them time to think about their answers. "I like being called Federico or Profe. I DJ. I am excited and a bit nervous, because I am going to try many new things in class this semester, and although academic tradition dictates that a professor is supposed to appear invulnerable and in control, I plan to put us in learning experiences that will not really be under my control."

My students said they loved making music, dancing, designing, playing video games, trying to solve crimes where the suspect was wrongly convicted. They were mostly excited and nervous, like me.

I explained my vision for this exercise: I wanted us to say these things out loud to remember that mathematics is a human endeavor. I hoped to make clear that our full humanity was not only welcome here, but in fact would define and enrich our mathematical space.

7. Quítalo del rincón.

After these introductions, I played a song:

`http://math.sfsu.edu/federico/Talks/embale.m4a`

In case you are not able to hear it, let me try to describe it, certainly not doing it any justice. The song starts with a pair of sticks, three drums, and a shaker, weaving an intricate combination of rhythms. A singer chants a long, melodic *aaalalalalalalalaaa*. Then he is joined by the main singer; in a beautiful and mysterious harmony, they introduce the theme of the song. The chorus comes in: a joyful call and response between the lead singer and a group of high-pitched voices—kids, maybe. While the kids keep

repeating the chorus, the main singer starts improvising rhymes, and the drums just take off. If you have played in a group like this, you will recognize the feeling: you stop knowing exactly what it is that you are doing, and you collectively connect to something deeper than anything you can reach on your own. After a couple of minutes the recording fades out, but you can tell this is just for technical or commercial reasons: the musicians show no sign of slowing down; they are only getting started.[1]

As the music played, I asked students to come up with a mental picture of what was happening, and write down a few words to describe it. I'll invite you to do that as well. Because students who could understand the lyrics were better prepared to answer this question, I asked them to step back for a moment and let the others answer first. They said:

community . joy . polyrhythm . family . crescendo . playful
encouraging . unexpected . churchlike . inviting . dancing
conversation . courage . motivation . cheerful . Spanish
learning . rhythm . celebration . style . culture . festive

Eventually, the Spanish speakers in the class explained the lyrics to everyone else: The song is a math lesson! *Quítalo del rincón* by Carlos Embales y los Roncos Chiquitos is a *guaguancó*; this is a style of Cuban rumba native to the Black neighborhoods of La Habana, born soon after the abolition of slavery in the late 1800s. The chorus says:

If someone doesn't wanna learn, we'll teach them, very happily!
20+3? **23**. 30+6? **36**. 20+3? **23**. 30+6? **36**.
Bring them out of the corner, towards the window;
you'll see how they'll learn right away, full of joy!
20+3? **23**. 30+6? **36**. 20+3? **23**. 30+6? **36**.

A few years earlier, on break during a mathematical visit to Ann Arbor, MI, I had found this enigmatic album for $1, digging through the sales bin of a music store—the dream of a record collector. It had been a great thrill to return home and hear this song! I still return to it often, when I think about what I'd like my math classrooms to feel like.

8. A community agreement

The course syllabus is the first official document students receive in a class; it is the first impression they receive about what is valued in the class. Twenty years ago, when I began teaching, I used to make the syllabus the night before classes started, essentially copying the syllabus from whoever taught the class last time—including the grading scheme. As an unintentional consequence, my class often valued whatever the last instructor valued. In recent years I have tried to write syllabi that actually communicate the kind of course that I hope to build together with my students.

That semester, after playing and discussing *Quítalo del Rincón*, we discussed the first part of the syllabus, which read:

In small groups, students discussed this agreement. To initiate a dialogue about it, I projected the agreement on the board, and asked each student to underline two words that particularly resonated with them. The wide variety of different answers was striking to me. Some students were excited that they would be challenged; some

[1] A recent performance of this song is at `https://www.youtube.com/watch?v=FpxE_xzPNQY`.

math 420/720 . combinatorics

spring 2017 san francisco state university

Community Agreement. This course aims to offer a joyful, meaningful, and empowering experience to **every** participant; we will build that rich experience together by devoting our strongest available effort to the class. You will be challenged and supported. Please be prepared to take an active, critical, patient, and generous role in your own learning and that of your classmates.

that they would be supported; some liked the combination of the two. Many liked the word "available"; most of them had jobs and many other obligations aside from being students, and they appreciated that this was acknowledged. We discussed how to be productively critical of each other's work, and what generosity might mean in a mathematics classroom. We talked about how sometimes we are very good at being patient with our classmates, but we are not so good at being patient with ourselves.

My students and I have cocreated this Community Agreement over the last few years, in collaboration with the organizers of the Encuentro Colombiano de Combinatoria (ECCO) [**FA**, **AB**]. Each semester, students start with the one used in the previous semester, and they make some (usually very minor, always thoughtful) changes. I then incorporate these changes into future agreements. I think it is important that this does not feel like an externally imposed code of conduct that they must obey. Instead, my hope is that we can reach a community agreement that is actually ours, that we are all excited to put into action.

9. Assessment.

To conclude the first day of class, we discussed the assessment scheme.

Grades in math classes are often largely based on exams and, to a lesser extent, on homework. This disproportionately rewards a certain kind of mathematician who enjoys and thrives solving problems quickly under pressure and time constraints. I am that kind of mathematician, and so are most current math professors, I suspect. After all, we had to succeed in this kind of grading system to become professors. But if we are honest with ourselves—and I am honest with my students about this—this is a very narrow kind of assessment; one that is easy for us to grade, but is not actually very good at measuring the kind of deep, creative thought that we associate with valuable mathematical work.

> We value what we measure because we do not know how to measure what we really value.
>
> Richard Tapia [**RT**]

Over the years, in the various classes that I have taught, I have searched for assessment methods that truly support student learning. I have learned immensely from my colleagues in math and science education, in particular Kim Seashore and Kimberly Tanner; I have found the survey [**ST**] quite useful. I have also asked students what

kinds of assessment might most accurately reflect the mathematical work they are capable of. As a consequence, I have shifted to grading schemes that promote student collaboration and reward many different types of mathematical work and communication.

For this paired undergraduate/graduate course, the assessment consisted of daily notecards (in pairs) summarizing the main lessons learned each day, homework, group reports summarizing the findings of the frequent group work activities, optional investigations on open-ended problems, a small research project (preferably but not mandatorily in groups), and a final five page "diary" summarizing the main lessons learned in the course. Collaboration was encouraged throughout all activities, with acknowledgment. There were no exams; I have always found combinatorial reasoning especially difficult to come up with on the spot.

I did not grade on a curve since I wished to promote collaboration. When I saw students behave competitively, I reminded them that they were not being compared to one another, and that the ultimate goal of the class was for every one of them to succeed. This was backed by research: Studies have shown that student learning in science classes is better supported by collaborative rather than competitive environments, particularly for women (who comprised 42% of the students in that class) and students from underrepresented minority groups (who comprised 42% of the students in that class) [**ST**].

10. From abstract goals to concrete practices.

On Day 1, my students and I generally concurred that our Community Agreement, and the words they used to describe *Quítalo del Rincón*, were good goals to aim for in our class. They felt that

community . joy . polyrhythm

were especially important. With that in mind, I asked them to propose a few concrete practices that we could follow to build this classroom culture and atmosphere together. They wrote them in notecards and brought them to class on Day 2, anonymously if they preferred.

They had some suggestions for me, the instructor; for example:

- Offer many group assignments where we get to work with different people each time.
- You told us that the course will emphasize growth and teamwork. Have the assessment and the grading reflect this.

Students also had many suggestions for themselves and each other:

- Let's be very mindful of how we communicate with each other. Emphasize constantly that mathematics is often difficult, and understanding is developed through extensive practice. Replace "this is obvious" with "with a bit of thought one can understand this"; "I'm stupid" with "I'm struggling"; "I can't do this" with "I can't do this yet."
- Let's not take the joy of discovery away from others. If I think I understand something, I should step back for a moment, and offer myself as a resource to others as needed.

- Let's stay honest and vulnerable. If I don't think I understand something, I should ask for help.
- Let's be excited to help our classmates learn, with some leadership from the teacher.
- It was so interesting how every instrument plays a totally different rhythm but altogether they create a very beautiful piece of art. Similarly, every brain works differently, and creating a math community to solve problems will make learning much fun, and will lead to more creativity.
- In that *guaguancó* we can only hear the musicians, but we're pretty sure the community is dancing right in front of them. Try to accomplish that in our class.[2]

All of these suggestions became part of our course syllabus, and they shaped our behaviors in the classroom. In particular, I ran the course very interactively with relatively little lecturing, frequent group work, and many opportunities for students to share their work in oral or written form with each other and with the whole class. The articles [**KT**] on classroom structures and [**BBDLW**] on active learning in mathematics provided several useful ideas. We maintained an open classroom policy; students and I brought our partners, family members, friends, and colleagues, and we did our best to ensure that they could participate in a meaningful way.

It is important to emphasize that student belonging and community cannot be arrived at by simply doing a nice activity on the first day of class; they must be nourished constantly throughout the semester. Although we did this in several ways in our classroom, in retrospect, I wish that we had discussed our classroom practices again at least once or twice during the semester, and that I had asked students for feedback on their in-class experience throughout the class, including the option to do so anonymously. This would have helped ensure that our experiences aligned as much as possible with our goals, and would have helped me intervene when they did not. I have since instituted a second part of the community agreement that can better hold us accountable; see [**AB**, Section 3].

The concrete practice that we most often returned to was the following:

- **Make space, take space.** If I feel comfortable speaking out, I should be mindful of how much space I take, and make room for others. If I tend to be quieter in groups, I should remember that my ideas are important, and others will benefit from hearing them.

This is easier said than done. Building trust is a prerequisite, and that is what some of these early semester activities tried to accomplish; but it is not enough. Megumi Asada and Pamela Harris [**AH**] offer valuable insights on how to help different students find ways to take space in the classroom in ways that are sensitive to and support their needs.

11. Make space, take space.

This last practice feels really relevant to me as I write this.

As a mathematics researcher with more than 20 years of experience, I feel pretty confident that my mathematical ideas are valuable. It sometimes takes a special effort to truly listen to students' ideas without projecting my own views onto them. When

[2] I still think a lot about what this might look like.

I have been able to really make space for students' thought, we have all learned very innovative and useful ways of thinking about combinatorics.

As a mathematics educator with great interest but under 20 minutes of formal training in education, I still feel like a student with everything to learn. Writing about pedagogy feels very uncomfortable. For every criticism the reader may have of my educational work, I have at least five. I cannot count the number of self-deprecating statements I have edited out of this note.

However, I did commit to upholding our Community Agreement. Now that the editors of the volume have made space for my thoughts, I feel compelled to embrace our collective cultural practice, take space, and speak-while-uncomfortable anyway.

12. Difference.

> Advocating the mere tolerance of difference [...] is a total denial of the creative function of difference in our lives. Difference must be not merely tolerated, but seen as a fund of necessary polarities between which our creativity can spark like a dialectic. [...] Community must not mean a shedding of our differences, nor the pathetic pretense that these differences do not exist.
>
> Audre Lorde [**AL**]

13. Music.

After our first meeting it occurred to me that, if I was asking students to help me create an ideal atmosphere for our class, then I should not be the only one choosing music for us; so the first homework read:

> Homework 0. Let's continue playing some music before class, to bring some more light into the classroom. On your designated day, please choose a song to share that makes you feel comfortable, joyful, at home. If you'd like to, you can tell us a bit about the song or why it's meaningful to you.

I emphasized to students that there was no obligation to share personal stories, and I also invited them to share something other than music if they preferred to.[3] We did this throughout the semester.

B.[4] got us started, playing a live performance of يا بحريه شدو ألهمه by Marcel Khalife; in the chorus, a stadium full of people sings "Oh freely, hey hey hey hey". She told us that as a Palestinian woman and an immigrant in the US who was simultaneously raising four children, working, and going to school, she found it very difficult to feel at home, welcome, and free in this country. But in mathematics, she found a place where she feels free, where no one can take her freedom away.

B. gave the class a three-song showcase of cumbia's migration from Colombia to Mexico to California. "Every Californian should know about cumbia."

F. and M. and X. shared songs they liked, with no explanations.

[3] In particular, an activity like this would have to be implemented very differently in classrooms with students who are deaf or hard of hearing.

[4] The students' initials have been changed throughout the paper.

U. chose *Dear Mama* by 2Pac: "My mom worked incredibly hard to give me the opportunity to go to college; when I'm in these classrooms, I am constantly thankful to her." This clearly resonated with several students, singing along.

D., a software engineer turned mathematician, shared the music and the journey of software engineer turned singer-songwriter Vienna Teng.

C. and D. and K. shared their favorite songs to perform.

D. told us that she wanted to share the song she sang at her mother's funeral. We did our best to hold space for her.[5]

K. played Lauryn Hill; "Who doesn't want to hear Lauryn Hill?"

N. made sure we knew that the Filipino-American hip-hop scene in the Bay Area is still going strong. Some of us knew Rocky Rivera as an MC, but none of us realized that she was a student on our campus.

Instead of a song, W. showed us a video of her son learning how to add; she told us that home is wherever she can be with her child, taking care of him.

B. brought her daughter to class one day. While young N. was on the board finding Eulerian paths in graphs with the rest of us, B. played N.'s favorite song: Israel Kamakawiwoʻole's *Somewhere Over The Rainbow*. It assured her that all her dreams can come true.

I was sure that my students would bring lots of good music, but I never imagined how deeply personal this exercise would turn out to be. One thing seemed very clear to me: my students wanted to be seen, really seen, as full humans, inside the classroom.

14. How I experienced this classroom.

Throughout my career I have tried to make my mathematics classroom a human place, where every interested student feels at home, and finds a conducive environment to discover and shape their own mathematical voice. It's a tall order, and I certainly will not claim that I have succeeded.

I can comfortably say this, though: That SFSU Combinatorics class felt like no other that I'd ever experienced. Teaching and learning in it was a tremendously human experience for me. Additionally, and relatedly, this was also the home to the richest mathematical discussions I had ever seen in one of my classes.

Let me confess something. When I devote a whole class period to getting us to know each other, when we spend a few minutes of every class sharing music that is meaningful to us, when we spend most of the time in each class period exploring mathematical situations together and at most 15 minutes "delivering content," I start worrying: Am I covering enough mathematics?

I have come to understand that when students are engaged so actively, and when we really listen to each other's ideas, a creative, mathematical magic can happen that I could not have arrived at by simply preparing a lecture and delivering it. In this class, more than ever before, I experienced my students truly take charge of their shared learning experience, take ownership of the material, allow themselves to ask their own critical, insightful mathematical questions, value those questions, and turn them into their own original discoveries. In fact, their insight taught me many new things about

[5]I use the term *holding space* to mean supporting someone by being fully present for them to process their feelings, without letting one's own feelings, ego, or proposed solutions interfere with that process. I now think it is a good complement to the suggestion of making space and taking space in the classroom.

classic problems that I thought I understood completely. More importantly, it led to new discoveries that I think only they would have come up with.[6]

I cannot take credit for this. In fact, I am certain that I will not be able to replicate it: a unique combination of humans made this classroom what it was, and led to a unique atmosphere and a unique mathematics. As a professor, I can only try to put some structure in place that may help my students and I flourish together. I continue to do this, each semester, with varying success.

15. Humanity.

> Teachers cannot claim their pedagogy is rehumanizing without obtaining recurring evidence from their students that they agree and without giving students opportunities to offer additional approaches for rehumanizing.
>
> Rochelle Gutiérrez [**RG**]

16. How (some) students experienced this classroom.

The following is a representative selection of students' feedback in the anonymous final course evaluations.

- The first day of a class wasn't spent reading through the syllabus or diving into material. Rather, it was spent entirely on introductions and conversation, setting the tone for a class in which students are deeply valued as human beings rather than just as mathematicians.
- The math was great, but the thing that stood out to me was the music. As I have been teaching for now 4 years, I try to continuously find different ways to make students feel comfortable/motivated/etc. [...] Having everyone have a chance to express themselves in that way in the class was awesome, so awesome that I actually used it in my class this semester.
- I am totally stealing classroom structures used this semester to implement in the classes that I will teach in the future.
- These are the kind of classes that remind me why I love math. I really enjoy the learning environment that was created. [...] I am not a fan of group work with preassigned groups. I would keep getting in the same group with an individual who's learning style was less than compatible with my own which was frustrating and a bit unnecessary.
- He tries really hard to engage with everyone and that paradoxically means that he doesn't have a lot of time for an individual student sometimes.
- I'm typically one who doesn't speak out much in class but working in groups helped me to become more comfortable and I found myself sharing more than usual.
- He ensures that each class member knows their opinions are important and that their voices should be heard. We established a supportive atmosphere and frequently worked in groups on difficult and interesting problems making sure everyone made a significant contribution and had a strong grasp on

[6]The mathematical work of this classroom will be the subject of an upcoming paper.

the material. I found myself pouring my extra time into this class because of how much I enjoyed learning the material we covered.

- As a combinatorics enthusiast, I have seen or self-discovered all problems/techniques covered in the class. I cannot afford to spend my sharpest years not learning, especially if I want to contribute to combinatorics. Each day, I hoped for something new, but each day was disappointing. I really did enjoy the homework, but I stopped attending class. Instead, I read the book at a higher rate at home and self studied upon completing the book.
- Math departments can be inhospitable elitist places where undergraduates who are earlier in their careers are looked down upon for not immediately grasping concepts. From the first day of class he builds a supportive environment for those students who may feel "non-brilliant" and helps them see that they have just as much to contribute as other students. He is always willing to seriously consider a student's ideas and suggestions. All of this while still pushing each of us to challenge ourselves and providing ways for students to pursue their specific interests.

It is important to say explicitly that not every student's experience was reflected in the comments above (74% of them filled in the final evaluation) and that course evaluations never tell the full story of the student experience. I shared a draft of this paper with my students, and was very grateful to receive additional feedback from some of them, including the following, shared here with the students' permission.

- [This paper] does not reflect my experience in your classroom and lacks nuance about who gets to safely bring their humanity into the classroom. I faced more sexism and microaggressions in your classroom, than anywhere else during my time at SFSU, but this might just be because I've learned how to deal with it now. The more feminine I was, the less I was taken seriously. You'd never notice. As a woman, I do not get to bring my humanity into the classroom and succeed.

  Even though that class may have been awful for me, you're onto something really important with the way that you teach. You were the first person in the education system who acknowledged to me that we do not all have the same privileges. I have my own classroom now, and do my absolute best to create a space where my students can engage in mathematics without having to "check themselves at the door," as you had put it.
- Thanks to you because you were (and always are) putting a great effort to create a welcoming environment inside your classroom. We know that you were doing this from your heart; I was able to sense that. Students are sensitive to their teacher's energy. Being different, I always felt comfortable inside your classroom; ego or use of power were never practiced inside or outside your class.

  I am reading "Culturally Responsive Teaching and the Brain" by Zaretta Hammond now. The author discusses that unwelcoming environments may cause real damage to [students'] brain cells which negatively affect their brains' intellectual capacity. Teaching is a very sensitive job. Teaching mathematics is even more serious.

Teaching mathematics is a serious job indeed. We followed these messages with long conversations that I am still learning from.

This feedback reminds me of Rochelle Gutiérrez's insistence that we do not romanticize the rehumanization of mathematics. Humanity comes with power dynamics, biases, and blind spots—including the teacher's—and it is heavily influenced by the effect of the structural inequalities of our societies. By breaking away from the traditional and more impersonal style of lecturing, and instead having everyone in the class interact constantly with each other, we are also making space for many new human challenges. How do we deal with them in useful ways?

Zeus Leonardo and Ronald Porter offer a framework for race dialogue that may also be useful for mathematics educators. They argue that a critical pedagogy is inherently risky, uncomfortable, and unsafe; it's impossible to create a truly safe space when the "violence is already there" [**LP**], explicitly or subtly. Instead they propose welcoming risk, since contradiction and tension are necessary if we are to really address the problems at the root of our societies and educational systems.

This has to be done with thought and care. Many students arrive to our classrooms despite significant structural barriers, and sometimes rather traumatic experiences within and outside of mathematics. As Shawn Ginwright points out, the work of helping them understand and overcome those structures and experiences is the work of hope and healing. In [**SG**] he studies how teachers and activists in urban schools use collective organizing and healing strategies to build hope and create social transformation. He highlights the central role of culture and identity, and the reciprocity between the teacher and the students in the healing process.

This work is rooted far from mathematics, but I believe it contains valuable lessons for mathematics educators. I am still seeking to understand how these strategies might be used productively and responsibly in our classrooms.

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Most importantly, I would like to thank my wonderful students; they are my teachers. Their diverse, critical, and generous perspectives have completely transformed my world view and my understanding of our work as educators. Working with them keeps my spirit young and my heart full.

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Congressive Question Time

Eugenia Cheng

It has almost become cliché to lament about narrow and off-putting stereotypes about mathematicians that lead many to feel like they wouldn't fit in in the mathematics community. But arguably a more pervasive signal that some might not thrive as mathematicians is the way mathematics education tends to cater to a particular personality type. Elementary students race their classmates to complete a worksheet of multiplication problems. Extra-curricular mathematics enrichment at the middle and high school levels takes the form of mathematics contests. And participation in the classroom typically demands an individual to offer an answer in front of a room full of their peers that might well be "wrong."

Such pedagogical structures cater to the ingressive student, who likes raising their hand and striving for individual accolades, who is motivated by competition, reassured by the thought that solutions are either "right" or "wrong", and feels secure enough to risk the potential for embarrassment. Ingressive students are more likely to be noticed by the professors at the university and offered special mentoring opportunities. They tend to excel at and enjoy standardized tests like the math subject GRE or do well at specialized exams like the IMO or Putnam that lead to admission and "competitive" graduate programs.

By contrast, congressive students prefer to work on open-ended problems in groups, for a much longer period of time. They're less likely to ask questions in class but more likely to make invaluable contributions in peer study groups, expressing their ideas in a way that doesn't make the person who is trying to learn something new feel stupid. The skills possessed by congressive students might be less likely to be noticed by their instructors but would later serve them (and their colleagues) well in an increasingly collaborative research environment.

— Emily Riehl

These are ideas I have been gradually developing across my whole mathematical career. I began as a grad student in the extremely male-dominated Cambridge department, moved around the world for post-doctoral positions, as is typically expected of ambitious young mathematicians, and won tenure in the ingressive environment of a "research led", "Russell Group" British university. But I was deeply unhappy, for a wide range of reasons including issues related to my identity (young, female, non-white) and issues more related to character and behaviour. I turned out to be powerless to address either of those issues, so moved into my current portfolio career teaching part-time at

the School of the Art Institute of Chicago (SAIC), while also writing and speaking extensively for general audiences, and performing concerts as a pianist. Like many art schools, SAIC is very female dominated, both in the student body and in management positions. Art students are also predominantly people who have been put off math in earlier levels of their education. I saw parallels with my experiences, although I was put off mainstream math at a very much later point in my academic career.

I came to see that some of these issues of exclusion are to do with identity, in terms of gender, race, and any number of other dimensions, but some are to do with character, and that conflating character with gender is mistaken and obstructive. I developed this theory and this terminology to help me understand my own experiences, to help me help my students, and to try and help the world escape the one-dimensional gendered thinking. I believe we need to address gender issues when they really are about gender, but not conflate issues with gender when they're really about character. This led to my book "X+Y: A Mathematician's Manifesto for Rethinking Gender" [**C**].

Recently I have given a number of talks introducing these ideas—such as at the Workshop on Professional Norms in Mathematics—and I have been increasingly struck by the ingressive structure of a talk followed by question time.

My thoughts on ingressive and congressive character traits are by nature a work in progress, as I continue to seek ways to make math and math education more congressive. In this article I would like to provide some updates. I'd like to start by describing a format of congressive question time that I developed as inspired by the workshop, for situations when a congressive presentation itself is not feasible or desirable. I will conclude with some remarks on virtual teaching, which most of us necessarily suddenly had to adapt to in 2020, and my thoughts on leveraging it to congressive ends.

Ingressive question time. During my talk it suddenly struck me that I was giving a talk in a manner completely unlike how I would ever teach any more: I was standing at the front talking, and there was going to be a raised-hands question time at the end. Asking a question in front of a group requires someone to believe that their question is worthwhile enough to take up people's time with, and for them to have the confidence that they won't be derided or belittled for their question. In my experience, the belief someone has that their question is worthwhile is largely independent of how worthwhile their question actually is, and correlates more with how straight, white and male they are; obviously this would require some research to test it thoroughly, but I would take the null hypothesis to be that there is no correlation with the actual worth of the question.

As for the second point, I would rather not frame this as confidence because then the onus seems to be on the potential questioner to have confidence or not. Rather, if someone fears that they will be derided it is probably a genuine risk, not just something they have invented in their head. They have probably been derided and belittled many times when they have tried to ask questions in a group—as so often happens to women, people of colour, and other marginalised groups. Moreover they are probably more sensitive to derision than those with more social capital (eg straight white cis men), just as people with less financial capital are more sensitive to crises such as a pandemic and global lockdown, where rich people are quite able to withstand it.

In a classroom setting I take careful steps to neutralise both of these issues. I make sure to explicitly nurture a congressive environment from the start, and to value congressive behaviour and not allow ingressive behaviour. I value all congressive contributions and make it clear that I will not tolerate derision or belittling of anyone. To neutralise the need to consider one's contribution worthwhile, I draw students' names out of a hat (mug) in turn throughout every class session, and express an interest in hearing what they are thinking. To reduce the pressure, I give them a chance to discuss it with their friends first, and I walk round the room joining in to assure them that their contributions are interesting.

This is all very well in a semester-long course, especially with the three-hour class sessions I have at art school, and class sizes capped at 20. There are several problems with doing this in a one-off "talk" time setting.

- Scaling up to large audiences.
- Having an audience of strangers where there is no time to develop and monitor congressive behaviour.
- A one-off intervention (a talk) often being more about conveying a large quantity of information rather than developing thought processes.

I will take these points in reverse order. For the last point, if I attend a talk by an expert it is quite likely that I want to hear as much as possible from that expert, not from the other audience members. For example for a research talk at a conference, it is an intriguingly bizarre mental exercise to imagine presenting one's research by inviting the audience to discuss it in groups and find a proof themselves, in real time.

With an audience of strangers the problem is that asking them to discuss things with each other might sound congressive, but has the danger of just giving the ingressive people even more chance to flourish to the detriment of more congressive people. When I attend workshop-style talks with more progressive speakers who ask audience members to turn and discuss things with each other, I am typically frozen in fear of my neighbours, and often with good reason: on the occasions when I have dared to speak to them, someone has invariably taken the opportunity to belittle and/or deride my thoughts. This happens even at workshops on inclusivity. I suspect that the desire to bolster one's own self-esteem by belittling others is simply too deeply rooted. I would also note that this fear is particularly justified for minorities, who are always at greater risk. And although Asians don't count as an underrepresented minority in math, we are still a minority; I have only twice seen another Asian woman at a conference, and have never in my entire life seen an Asian woman give a talk.

Scaling up to a large audience is another problem, as with an audience of a hundred it is already difficult for a presenter to go round engaging with all the small groups in the room, let alone an audience of a thousand or more. The larger the audience, the more difficult it is for congressive people and those who suffer real social threat to participate. One of these issues is about character traits and not specific identities, but the other is about specific identities as there is genuinely more social risk for women, people of color, and LGBTQI+ people in these spaces. A typical hands-raised question time is, in my experience, entirely dominated by straight white men, or people who pass for such. And yet afterwards, for example if I am signing books, many women, POC and congressive people come and ask me genuinely interesting questions prefaced by the fact that they didn't dare ask the question in public.

Here are some solutions that I have seen proposed or used, and the problems I have seen with them.

Stern rules at the start. A chair might begin question time with stern admonishments such as "Your question must be only one sentence" or "Please remember that a question ends with a question mark". However, sticking to one sentence questions eliminates the friendly practice of saying "First I'd like to thank you for your wonderful talk. Here's my question:" and doesn't stop people ranting for five minutes about their own theories in one long unpunctuated sentence. And those same people can always end with a question mark simply by appending "And my question is: what do you think of that?"

Asking for a woman first. Some research has shown that if a woman asks the first question then there is greater participation of women in question time. The trouble with this practice is that it is divisive, or rather, it makes some entitled men very angry as they think they are being discriminated against. I tried it one time and all it meant was that the next question was from a man who was so enraged by that practice that he yelled at me for even longer than he otherwise would have done.

Taking questions on post-its, or in an on-line forum. This is a decent idea, as the anonymity means that people can be more daring about the questions they ask. Unfortunately it means everyone is more daring, including the obnoxious people. I have had questions in anonymous online forums that I can't imagine someone asking if they had to stand up and say it in person. The issue is then exacerbated by the fact that I can't read their body language, nor can I look them in the eye in my response.

With many of these proposed solutions we run into what I call the problem of "antibiotic-resistant superbugs". That is, we eliminate the mildly obnoxious people and leave the extremely obnoxious people to dominate even more strongly. I wanted a solution that would neutralise the ingressive people without making them feel singled out or discriminated against.

Congressive question time. Here is a format for congressive question time, which I first experimentally tried out during an informal presentation in the math department at the University of Wisconsin, Madison. It was a particularly conducive environment, with a small audience in a common room. It was also a situation with great potential benefit not just for congressive people, non-male people, and minorities, but because the audience consisted of professors, graduate students, and undergraduatess. In such settings it is very daunting for undergraduates to speak up.

The format is that I do not take any questions with raised hands, but instead I invite people to hold discussions about my talk with people around them if they like (or not if they don't) and I wander round the room joining in. Having prefaced it with the reason I am using this format, I make it clear that the idea is to increase inclusivity and hope that this will help them focus on congressive conversations rather than ingressive show-downs. As I walk around I can particularly make eye contact with women and visible minorities and see if they would like to ask anything; also I think it is much less daunting to speak to me quietly without everyone else hearing, and congressive people are much more likely to interact with me as I go round. It's also a pleasant experience as people often simply express appreciation without having anything to ask, and those expressions are not really allowed in a hands-raised question time.

But moreover, ingressive behaviour seems to be neutralised. The format does seem to discourage people from just yelling at me angrily, as we are now face to face having a quiet conversation, and perhaps they are demotivated by not having a large audience listening to them. Of course, this is not a large randomised trial, but I ran this sort of question time at every talk I gave until COVID-19 ended group gatherings for the time being, and it was a very congressively positive experience every time.

I take notes as I go round, and then I finish the session by going back to the front and sharing a summary of the discussions that we had. After the first successful trial run I scaled it up to a public talk at the same university, and have since used the format with very large audiences even in auditoriums with fixed seating. It's true that I can't talk to everyone in the time available if the audience is large, but I can interact with many, many more people than in a raised-hands question time, with a large skewing towards congressive people without me having explicitly excluded anyone for their identity.

The development of congressive question time was going ahead apace when all talks and much of life suddenly went online as protection against the global Coronavirus pandemic of 2020. There is much to be said about ingressive vs congressive responses to the virus at every level from personal behaviour (whether wearing a mask is seen in relation to personal risk or protection of the community) to government responses (whether to focus on protecting ordinary citizens or attacking and blaming other countries, for example).

I just want to focus on my congressive classes and what happened to them during the sudden and unpremeditated switch to online teaching.

Virtual teaching. At SAIC we were around halfway through the spring semester when we went into lockdown. Classes were cancelled for three weeks to give us time to prepare for online teaching, and then the rest of the semester was online. This presented extraordinary difficulties for studio classes in an art school, but no insurmountable difficulties for me teaching math.

While online teaching is obviously far from ideal I tried to focus on getting the best out of it and ended up finding many real benefits as compared with teaching in person.

I'd like to preface this with some surprise: for some years one of my central principles in teaching has been to make sure I deliver something that could not be delivered by a video. (When I started teaching, my central principle was analogous but I only had to compare myself with what could be delivered by a book, as the internet existed but video-sharing was still in the future.) With this principle in mind, I wondered whether or not to be surprised that I could in fact pivot to online teaching with some success.

However, in the new era of enforced online teaching my principle was flipped: I wanted to make sure I delivered something that could not be delivered in person. I did not want my online classes just to be a poor approximation of classroom teaching, although I was greatly encouraged by messages urging us to "Do it badly", that is, remember that classes were at that point not the most pressing need of society and our students, and that we should not wreck ourselves trying to do it in expert fashion.

I opted to deliver the rest of my classes (five weeks) entirely asynchronously. This was not my original intention, but in training sessions we were very strongly urged to stick to asynchronous teaching for reasons of accessibility. Some students might not be able to access a computer, a quiet space, or wifi at the designated class time. They

might need to take on extra jobs to make ends meet, or be looking after family members. And many had been forced to go home to very far off time zones which would make our class time in Chicago highly disruptive to their sleep. I was convinced by these arguments and my congressive and inclusive teaching principles.

In my switch to online teaching I was greatly aided by the fortuitous fact that I had just that semester decided to start teaching from slides, using my touchscreen laptop to annotate the slides in realtime while projecting them in class. I made the full annotated pdf available to students before class so that they could take an advanced look if they chose, and then annotate on their own tablets in class. I also made the fully annotated pdf available afterwards. I have been resistant to the idea of providing notes but finally realised that we have to teach the students we have. Some students are good at taking notes and others aren't, and I don't wish to keep excluding those who are not good at taking notes, as they have already been excluded from the entire education system (but might do extremely well in a painting class, unlike me).

This meant that with a screen sharing video I could give my class in almost the same fashion, using slides and annotating them as I went along. However, a large part of my class format is typically discussion and group activities. Also, that semester was a math and music class so I also played the piano and sang in class to demonstrate pieces of music, and had the students compose during the class, implementing mathematical principles in the composition.

Some aspects of this were necessarily lost but some congressive things were also gained.

Removal of time pressure. Asynchronous videos enable students to watch repeatedly and go back over explanations if they need to hear it again, without being embarrassed to ask again in class. My experience of teaching art students is that there is an extremely wide range of rates of understanding, and so it is essentially impossible to explain something to a class at the same time. (In fact I have found this to be the case among math majors as well, and even more so in earlier stages of education). Thus I end up having to walk round the room to give the explanation again to those who just need to hear it again, who lost concentration, or were temporarily texting someone. (I don't object to students texting during class—I rather like texting during talks myself, so fully understand the urge.)

Many students told me that they watched parts of the video several times and understood it after several re-watchings. The videos also had congressive activities for the students to explore and I invited them to press pause and take some time to explore them. Here again they could take as much or as little time as they wanted, without being put off by seeing other people getting further than them more quickly, nor being tempted to compete to make faster progress than other people.

Removal of peer pressure. Students are relieved of peer pressure in general and don't need to worry about whether other people are picking things up faster than them. As my students are typically congressive students who did poorly in standard education, particularly math, they are likely to be particularly susceptible to peer pressure. I think this is partly because standard education specifically filters ingressive people in and congressive people out. On a related issue, a large majority of my students are

not straight-white-male-passing, so suffer high levels of social threat as well. This is all removed by them watching a video quietly at home.

Accessibility to different character types. Students can keep time in their own ways. I have nothing synchronous and some students go through all the materials in the normal class time and others do it later. I always find that some students are consistently early for class and others are highly variable, and it's hard to keep a balance of politeness to the punctual ones without being excessively authoritarian about time keeping. I don't see why the ability to keep time should be used as a filter to keep people out of mathematics. After all, I know plenty of successful research mathematicians who are terrible at time keeping and deadlines, sometimes because they are so engrossed in research that they lose track of the world.

I was happy with the relaxing of standards and greater flexibility. As students were going through the materials in their own time, it also felt appropriate to let them hand in work in their own time; by contrast when we're covering particular material in a particular class time I try to get them to keep up with the work so that they can follow. But with some students that is a bit like pulling teeth, and it's not my favourite aspect of teaching. It feels ingressive and very far from pedagogical principles.

Encouragement of valuable transferrable skills. Some students shared that they were finding it harder not to be lazy—they had to be self-motivated and make their own work ethic and schedule. While this was harder for some of them than others, and especially hard as it was somewhat new, I do think these are genuinely important life skills, so I embraced a method of teaching that nurtured those skills as a side effect.

Congressive discussion boards. The interaction we had was on text based discussion boards. Some students told me it was too daunting to post in public so they messaged me privately. Many students expressed heartfelt appreciation on the discussion boards, and I did the same in return. It somehow felt easier to do that on the boards than face to face, and this was pleasant all round.

Overall I was glad to hear from students that while the class has moved smoothly to online, it was not the same as being in a classroom together. I'm glad to hear this because I have always aimed to give more in a classroom than would be possible in a video, and so if they said the video was just as good it would show I had not succeeded in my classroom aims. Some students have previously asked for classes to be videoed so that they can watch if they can't make it, but I am strongly averse to simply filming a class. An in-person class and an online class are two very different things, just as a good movie is not just a stage play that has been filmed (although those can also be interesting). For that reason I would never want to give a class that some students take in person and some take virtually.

However, I do see that for some students a virtual class is highly beneficial, for example:

- housebound students
- students who are fitting in studies around work, parenthood or other carer roles
- students who do better with flexible schedules
- students who experience anxiety in a group situation, but who are motivated to learn

- students who are intimidated by other students' behaviour, especially those with social disadvantages on grounds such as race, sexual orientation, gender presentation, socio-economics, previous education history
- students who are just generally quieter and/or more congressive.

In fact, I think I would have done rather well with virtual classes, especially instead of large group lectures which invariably went too fast for me and in which I never, in my whole student career, dared to ask a question. I see that the University of Cambridge will be giving all its lectures by video for the coming academic year, and only small group tutorials will remain in person. I advocated for this sort of approach for years at the University of Sheffield, for reasons of accessibility, inclusivity, and better use of staff time, but all to no avail. They were congressive reasons, but I didn't have that terminology at the time. Nor did I have enough influence to effect any change at those ingressive insitutions; it seems to have taken a pandemic to do so.

I hope we can learn some more congressive ways from the disasters of 2020.

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Mathematics, We Have a Problem

Michelle Manes

DISCLAIMERS. My comments are informed by experiences as a Program Officer at the National Science Foundation, but any opinions, findings, conclusions, or recommendations expressed are my own and do not reflect the views of the NSF.

A note on gender and pronouns: Throughout this article, I write about harassment and discrimination of women by men, and I use pronouns (she/her and he/him) that are consistent with this scenario. I understand that this is not the only scenario, that women can and do sexually harass men and other women, that men harass and discriminate against other men, and that our queer and nonbinary students and colleagues are especially vulnerable to harassment and discrimination. However, I am writing about my story and my experiences, and I find this specificity helpful to me as the writer. I hope that no one feels excluded by the way I have written this piece, and I want to state clearly and unreservedly that I stand against harassment and discrimination in all of its forms.

18. My story

If I'm going to talk about sexual harassment and discrimination, I may as well begin with my own story. Like every woman I know, I've had many (too many) personal experiences with sexual harassment throughout my life. This is a very short sample of a very long list:

- When I was quite young, a family friend we all called "uncle" used to grab at me and my cousins in the pool.
- An elderly next-door-neighbor would insist on a kiss a hello and then stick his tongue down my 12-year-old throat (my "first kiss").
- I've been groped on public transportation, flashed, and cat-called more times than I can count.
- Twice I've been driving in the car on a highway and noticed a guy driving in the next lane masturbating, positioned so that I can see everything, looking directly at me.
- I have been followed on multiple occasions. For example, late one night I was walking near the university. A stranger followed me for several blocks, reached out to grab my shoulder, and said, "I just wanted to meet you." Another time, during a conference in Austria, some guy followed me at lunch every single day for two weeks.

If you are a man, and if the women in your life don't talk to you about these things, then this might sound like a lot. But I want to emphasize how mundane these experiences are for most women, how typical this all is. Everything I've been through—though it started before I was ten and continues now in my sixth decade of life—is minor compared to what many women have experienced. I've never been raped or truly physically hurt. I've never been drugged nor have I feared for my life. None of this rises to the level that I've told my mom about it, much less the police. But this is my experience of the world; I have been viewed as a sexual object (and often not much else) since before I was ten years old. This is the low background noise of my everyday life, and occasionally it becomes a cacophony that can't be ignored.

Of course, I'm not just a woman, but a woman working in mathematics. It should come as no surprise that sexual harassment and discrimination has played a role in my professional life as well:

- I was the only woman in my undergraduate logic class. The third time I correctly answered a question in class, the professor reassured the other students (all men), saying that I was only good at logic because women like to argue. Even he, a world-famous logician, wouldn't argue with his wife because she always wins. (I never spoke up in class again, in that one or any other during my undergraduate years.)
- I was in the stacks in the math library. When I walked into a row, I noticed a guy looking at books on a lower shelf, and at some point I realized that he was no longer in my peripheral vision. I looked down and found him lying on the ground, head between my feet, looking up my skirt. I ran to the front desk of the library and told the student working there what happened. She replied "eeeewwwww" and did nothing more.
- Some of my professors rarely looked anywhere but my chest when talking to me.
- I've been hit on at the Joint Math Meetings several times.
- My department recommended raises for several men in the department, the outcome of which would have been that I (the only female Associate Professor) would have been paid less than every male Assistant and Associate Professor.

But is that really harassment? One of the difficulties I have in talking about my experiences of sexual harassment in academia is that none of my experiences are really *that bad*. I'm one of the lucky ones. No professor or TA ever solicited sex from me. My advisor never showed up drunk at my hotel room door while we were at a conference. No faculty member walked into my office and asked me questions about my preferred sexual positions or told jokes about masturbation while I was sitting there with my students. No world-famous mathematician asked me to call him "Daddy" while we were alone in a car. No colleague has pulled my hair or joked about my body. No faculty member dropped his pants at a party I attended nor professed his love for me when I just wanted to learn math from him. No faculty member has physically blocked me from leaving a room until I gave him a hug that I did not want to give. No mathematician has, upon our first meeting at a conference, asked personal questions about my sexual history over dinner while other senior faculty sat there, laughing about it. Have I *really* been harassed?

In [**5**], the authors identify three types of sexual harassment:

- gender harassment: "verbal and nonverbal behaviors that convey hostility, objectification, exclusion or second-class status,"
- unwanted sexual attention, and
- sexual coercion.

The first is the most common. It's easy to discount, but it's not unimportant. These experiences make me feel small, make me uncomfortable, and remind me in subtle and not-so-subtle ways that I don't belong. Gender harassment is the very direct reason why it took me ten years after earning my undergraduate degree to enter a graduate program in mathematics. It's worth noting that more egregious sexual harassment and assault happen in places where gender harassment is present and tolerated (see [**5**]).

A toxic environment. Until the past couple of years, the biggest impact of sexual harassment and discrimination in my professional life involved a woman I've never even met: Jenny Harrison. It's impossible to tell the story of me as a mathematician without talking about her.

The year before I arrived at UC Berkeley as an undergraduate, Jenny Harrison was denied tenure by the university. The year after I graduated, she won her long and drawn-out legal battle in which she claimed that decision was based on sex discrimination. Her tenure denial colored my entire undergraduate experience. Students who predated me by a couple of years talked about her in glowing terms. Of course, tenure decisions at a place like Berkeley are not based on teaching or on what students think. But the only tenured woman on that faculty of nearly 100 was Marina Ratner, again someone I never met but about whom I only heard negative remarks and gender-based slurs. I don't know Jenny Harrison's research well enough, nor Berkeley's tenure standards well enough, to know if she "deserved" to get tenure on her "merits" (whatever those words mean to you). But I can tell you that the women in that department—the undergraduate and graduate students walking around the building, attending classes and seminars, and working in the computer lab in the basement—heard what was said about the only two women on the permanent faculty, and we took notice. Many of us opted not to continue on to graduate school in mathematics. For me it was quite clear: someone like me was unwelcome in a department like Berkeley. Why should I even try?

More recently, I've worked in a department where several male faculty members dated their former students. In all cases, there was (as far as anyone can tell) no violation of Title IX or university policy. The relationships began after the student / teacher relationship ended—weeks, days, or even hours after, but still "after." The relationships were consensual and were (at least eventually) disclosed to the relevant supervisors. Some of my male colleagues were angry with me for arguing that there was something wrong with this behavior. (One or two of them got it immediately and were supportive. But not, by any means, the majority of them.) Of course, they did not have upset graduate students coming to them saying, "I guess it's ok to sleep with your students now?" Those students came to *me*. I had to answer for what was going on in the department, for what my male colleagues were doing and what my female students were feeling. Many of the men in my department couldn't imagine that there was a problem if no law or rule was broken, but I could.

I could imagine myself as a student in a Calculus 1 class, sitting next to the same student every day for the whole fall semester. And I could imagine returning to campus in the spring, seeing that student walking around the department arm-in-arm with the professor who had taught the class, seeing them kiss. I understand the message that sends to me about my role in a department full of male faculty and graduate students: I am a potential date, not a serious student or a future colleague. I can't trust a kindness or an offer for extra help. I come to believe that any attention offered to me isn't because of my mathematical abilities but because there might be some romantic connection down the line. I come to believe that professors are only supportive of me and my thoughts because they have ulterior motives.

When all of this happened in my department, several people thought a reasonable response to my complaints was to regale me with stories of couples for whom this "worked out," who are still together long after beginning a student / teacher relationship. But here's the thing: I don't care about their individual happiness. What you don't see, when you look at their decades-long relationship, is the women who questioned their position in the department and in mathematics. When you just look at the happy couple, you don't see the toxic environment their relationship created or the people who were harmed. But believe me, they are there.

Research again backs up my contention. From [**5**]: "Sexual harassment does not only impact the target but may also impact employees and coworkers who witness or hear about the experience…women who experience sexual harassment directly and indirectly report higher levels of absenteeism and intentions to quit, and are more likely to leave work early, take long breaks, and miss meetings (job withdrawal)." In fact, "all employees in the workplace—both female and male—can suffer from working in a climate perceived to be hostile toward women."

19. The institutions won't save us

During my career, I have come to recognize that the fundamental goal of any institution is to preserve its own existence. Much of how NSF functions (low funding rates, for example) makes sense through this lens. On my first day of training, I learned about the "*Washington Post* test": we should not say or do anything that would reflect badly on the agency, should our statement or action appear on the front page of the newspaper. The local paper is read by members of Congress, and Congress sets the NSF budget. The primary goal of the NSF is to maintain or grow its level of funding from year-to-year; everything else is secondary.

Title IX. During the 10 years I've been in my department, several students have come to me after experiencing sexual harassment. I've been through the required Title IX training and know that I am required to report this, so I did. But each time, involving the Title IX office made things worse, not better, for the students. They were re-traumatized by being forced to tell their stories again and again, this time to complete strangers instead of to a trusted faculty member with whom they already had a relationship. Each time, there was "no finding." Nothing got better for these students, and in fact everything got worse, as soon as the Title IX office was involved. I started to question the role of the Title IX office. I began to realize the truth: The Title IX office serves the university, not the students. Their role, in the surprisingly honest words of

one Title IX coordinator, is "minimizing conflict between students and the school," not enforcing the law and not protecting or helping victims of abuse (see [**10**]).

Talking with friends who work outside of academia, I found that this is how women in the tech industry view Human Resources (HR) departments. HR is not there to help victims of harassment and discrimination. HR is paid by the company, and their job is to prevent the company from having to deal with lawsuits by any means necessary. Sometimes that will mean taking action against abusers of power, but more often it will mean cover-ups and further victimization of more vulnerable employees. Never, my friends tell me, bring your complaints to HR.

I wasn't sure what to do with that information. My Title IX training told me that I was required to take students there when I found out about harassment. But I started to wonder: Required by whom? Under penalty of what? I'm a tenured full professor. What will the university do to me if they find out that I knew about sexual harassment and didn't force the victim to report to the Title IX office against her will? Did this requirement really have the force of law, as I was being led to believe?

For context: Before returning to a Ph.D. program in mathematics, I worked for many years in K–12 education. Teachers of young children are, by law, mandatory reporters. If they suspect a child is living in dangerous circumstances—is being abused or neglected—they must report. But not to an office of the school; they must report to law enforcement. The law requires it. I had to ask myself: Does a federal law like Title IX really mandate that I report to an officer of the university? That I report to an office with no legal standing, with no authority within the legal system, and with allegiance to the university rather than to the victim or to the law?

In fact, I learned that the "mandate" to report to the Title IX office and the designation of faculty as "Responsible Employees" is university policy, not law. I know what I have decided to do with this information, and I will let you decide for yourself.

NSF Term and Condition. On October 22, 2018, NSF became the first federal funding agency to require institutions to report findings of sexual harassment by Principal Investigators (PIs) funded by the agency. This new requirement—the "Term and Condition: Sexual Harassment, Other Forms of Harassment, or Sexual Assault" [**15**]—made news in scientific publications [**13**,**19**], was written up in mainstream media like the New York Times [**11**], and was heralded as a great step forward. NSF Director Dr. France Córdova, was lauded for her vision and her willingness to tackle the issue.

From inside the Foundation, however, I saw a very different story. I went to several trainings where the new "Term and Condition" document was discussed and where I could ask questions about it. By the end of my first year, it was clear to me: NSF has a policy on sexual harassment, and that policy is both toothless and unenforced. I was, and continue to be, shocked that no such policy already existed. I was, and continue to be, disappointed at how this policy was so celebrated by the scientific community as a great step forward. And I was, and continue to be, heartbroken at my inability to help the numerous women who call me, hoping that this new policy will help them deal with seemingly impossible situations.

When I got to NSF, I couldn't have imagined how much of my time and energy would be taken up with questions of sexual harassment by mathematicians. I couldn't have known that this "Term and Condition" document would come to define a large portion of my work or that I would become a fixture at the Office of Diversity and

Inclusion (ODI). I couldn't have known that I would have an email folder called "Harassment," nor how frequently I would add to it. I didn't know that friends, colleagues, and strangers would come forward with their own stories, hoping that this "Term and Condition" document meant that someone at NSF would listen, that someone would act.

The policy requires awardee organizations[1] to notify NSF of:

- findings or determinations that an NSF-funded PI committed harassment,
- placement of a PI on administrative leave, or
- the imposition of any administrative action relating to a harassment or sexual assault finding or investigation.

When I read the document, my first thought was: this further disincentivizes institutions from conducting honest investigations. I already know that Title IX offices have a goal of protecting the university from lawsuits, not protecting victims of harassment. They are already inclined towards non-findings, dragging out investigations until students give up or go away, and cosmetic fixes like changing a victimized student's class schedule. If action against an NSF-funded PI might mean the loss of grant money for a university, there is even more reason to take this "protect the institution" approach. And, in fact, in at least one case brought to my attention, a Title IX officer said to students who had filed a complaint, "Except for this [pattern of harassment of many graduate students over many years], he is on track for a world-class career," and then she determined that the PI's actions did not constitute sexual harassment.

The NSF Term and Condition requires the **authorized organizational representatives** to self-report **findings** of harassment. It relies on the flawed and biased Title IX office to conduct timely and honest investigations and to report them. It does not require reporting credible complaints that are under investigation, nor reporting repeated complaints against the same PI.

Even when reports are made, there is no clear path forward for the agency. According to the Term and Condition document: "NSF may, if necessary, initiate the substitution or removal of the PI or any co-PI, reduce the award funding amount, or where neither of those previous options is available or adequate, suspend or terminate the award." The key word here is "may." No action at all on the part of NSF is required. At every training I attended, the NSF staff from ODI were loath to promise any particular consequences. The message was, "We don't want to disrupt or negatively impact the science." I was left with the distinct impression that the new policy left the option that absolutely nothing would be done, and that in fact that was the most likely outcome of any report.

Despite being a woman at the helm of the primary funding agency for fundamental science and engineering research in the United States in the era of #MeToo, Córdova was an unlikely leader in this effort. In her own words [**1**], "My personality sort of shies away from that kind of thing because I come out of a generation that just sort of said 'oh well,'…We just would turn away from that sort of thing and [think] that there was

[1]For almost all NSF grants, the awardee is the *institution*—the university or the research center—and not the individual PI. Except in rare cases, a PI writes a grant proposal, and the institution's Sponsored Research Office (SRO) signs off on and submits the grant. Each institution has a negotiated indirect cost ("overhead") rate—often 50% to 70%—that is added onto the PI's request. This money goes directly to the institution. Ultimately, the SRO is responsible for abiding by all of NSF's requirements in terms of spending and reporting on grant activities.

nothing we could do about it. That's kind of how we felt, just to get on with it and to bury our heads in research and move on." In fact, she made clear her motivation for taking this action against sexual harassment [**13**]: "Córdova said then that she hoped the new policy would prevent the agency from being blindsided by media reports of NSF grantees who are harassers."

NSF's goal is not to protect victims of abuse; the goal is not to make science a safer place for women. The goal of this policy, plain and simple, is to protect the NSF from bad press, it's the "*Washington Post* test" writ large. This NSF Term and Condition is born from the same goal that drives HR departments and Title IX offices: institutional self-preservation.

20. So, what should we do?

This is where I have to reiterate: all opinions and recommendations are mine, and they do not reflect those of any employer I have ever had. But I, as a person with no power to implement any changes anywhere, offer some suggestions for what we can do as individuals and as a community. I also have some recommendations for our institutions, which I know will never be put into practice. But a girl can dream.

The straw man of false reports and the truth of ruined careers. Whenever I make suggestions for concrete actions to address and reduce sexual harassment in the mathematical community, I immediately hear how unfair it would be if someone were falsely accused and were subjected to these measures. If someone lacked NSF funding while an investigation was completed. If some innocent man's reputation were to suffer. So before I get to my recommendations, I want to get this out of the way: There is not an epidemic of false reporting, and until there is, I'm not interested in any conversation with that as a premise.

My experience tells me that false reporting of sexual harassment is vanishingly rare. The truth is that women don't even report the harassment that **does** happen, because nothing good comes of reporting. Harassment reports are (as my male colleagues enthusiastically remind me) "a he-said / she-said situation." There are rarely witnesses, and there is almost never "proof." As a rule, the harassers are senior, tenured, and protected. The victims of harassment are junior, more vulnerable in their career, in need of recommendations, and afraid of being labeled a troublemaker. I have never heard of someone's career benefitting from making allegations of harassment, even allegations that held up under investigation. Most often, the accuser's career suffers irreparable harm and the accused goes on after some minor inconvenience. In the words of Wendy Brown, a political scientist at UC Berkeley [**12**], "too many harassers in academia are never punished while their victims may be forced to alter their whole careers."

In one of the most well-known cases in recent years [**17**], Jane Willenbring was sexually harassed by her advisor, geologist David Marchant, during a 3-week stint in Antarctica. After her graduate study, Willenbring changed fields to avoid doing future work with her abuser, and she feared a severe impact on her career because of that change. She waited **17 years** to report the harassment, until she had a secure tenured position. After she reported, several other women came forward with similar stories. Ultimately, an investigation by Boston University found Marchant guilty. But even in this case, one of the rare cases with enough evidence to produce a finding, the victim didn't feel safe reporting and chose instead to leave her field of study.

Several articles [**8**, **10**, **14**, **18**] describe the case of Caltech astrophysicist (and 2012 NSF CAREER awardee) Christian Ott, who was accused by several graduate students of sexual harassment. Caltech was "absolutely rigorous about applying legal privacy protections to the perpetrator," but failed to protect the two women from retaliation [**10**]. After an investigation, Ott was barred from campus for just six months, then allowed back, allowed to work with graduate students, and allowed access to his two accusers. In the words of one of these women: "Because Christian still has a place at Caltech, I feel that I don't." In fact, she did leave, completing her studies at another institution. Ultimately, Ott was indeed fired, but not before much damage was done. The two students who filed the formal complaint suffered delays of at least two years in their degree programs, and at least seven left the research group in a short time.

Anecdotes are not data, but research backs up my impressions. Several studies on sexual harassment in academia have been conducted in recent years, and their findings vary in the details but not in the broad strokes [**5**]: Somewhere between 38% and 69% of women in graduate or professional school experience sexual harassment on campus, but only 3% to 10% report it to anyone at the university.

If you are more worried about the career implications for an accused harasser than the career implications for the women being harassed, then you really don't understand the situation. Fear of retaliation has made filing a formal complaint "the least common response" to harassment [**4**]. If there is no real or perceived benefit to reporting harassment, what reason would a woman have for falsely reporting?

Recommendation 1: Believe women. Out loud. Words cannot describe the feeling in my gut when a I tell a colleague—a "good guy"—about a personal experience with harassment and he asks "like, what did he do *exactly*?" I have to prove that what I experienced was actually "that bad," that it was really harassment as *he* defines it. My definition and my feelings on the matter are only valid if a man concurs, if he judges what I experienced to be out-of-bounds and not simply a misunderstanding or "a compliment." I don't know how to express how I feel when I mention a notorious harasser—about whom I've heard multiple, detailed stories—and the response is "allegedly," dripping with derision. Or "well we haven't heard his side." But I will tell you this: I know that anyone who responds in this way is not a "good guy." He's not an ally, and he's not to be trusted.

If you are present when a woman talks about harassment—whether it was her experience, or someone she knows, or something she heard about—don't express doubt. And if other men worry about the potential for ruined careers of abusers, talk about the lack of reporting of harassment, the fact that there is no incentive at all to file false reports and no evidence that this is at all common, and that the career implications are much worse for the victims of harassment. Ask them to name a single person brought low by false reports.

If you hear harassment rumors about the same person multiple times, there is something there. Women don't have much actual power in the world, much less in academia. Whisper networks are what we have. If women are telling you that someone is a bad actor, believe them and act accordingly.

Recommendation 2: Stop prioritizing men's entitlement to reputation over women's entitlement to exist without fear and humiliation. When victims come forward, they must be able to rely on our support. Do not assume that if the allegations were true that you would have seen or experienced the behavior. Harassers don't harass everyone. The way your colleague treats you does not tell you anything about how he treats other people.

Don't collaborate with harassers. Don't bolster their careers. Doing great mathematics does not excuse the harm. Prioritize people over science, even if your personal research agenda will suffer for it. Otherwise, you not only enable harassers, but you also send a devastating message to the individuals who have been harassed—and whose careers are often seriously disrupted or destroyed—as well as to the whole community. Don't make excuses. Don't focus on the letter of the law. Realize that if you enable harassers and if you accept toxic environments, there are real consequences.

Prioritize the safety of students and faculty, especially those from marginalized groups, over the reputations of "great scientists." I am asking this of each of you as individuals, but I will point out that it's even possible for organizations to do this. Take, for example, this statement released by Turku University, who fired Ott just a month before he was supposed to begin working there (Ghorayshi, 2018 [**9**]): "Our concern and solidarity is first with victims of harassment, and with the right of all staff and students to work in a healthy and safe environment."

Recommendation 3: Evaluate and overhaul tenure requirements and the Term and Condition document. The Term and Condition document was always meant to be just a first step, but according to the Government Accountability Office [**3**], all of the federal agencies that fund scientific research have established policies around sexual harassment "without a plan, and without methods to evaluate their policies and how they communicate them." NSF and the other agencies should immediately set a timeline for reviewing the existing policies—including listening sessions and a period of public comment—and a specific date for revised policies to be implemented.

Revised policies should treat sexual harassment like scientific misconduct. Because it is. I am certainly not the first person to suggest this. From [**5**]: "Federal agencies should…attend to sexual harassment with at least the same level of attention and resources as devoted to research misconduct. They should increase collaboration among offices that oversee the integrity of research (i.e., those that cover ethics, research misconduct, diversity, and harassment issues); centralize resources, information, and expertise; provide more resources for handling complaints and working with targets; and implement sanctions on researchers found guilty of sexual harassment."

Universities and departments should do this as well. A positive tenure review should indicate that the candidate is in good standing with the scientific community. A scientist being investigated for falsifying data or found guilty of plagiarism would not pass tenure review. The same should be true of known harassers. In most departments, tenure committees are only allowed to evaluate a candidate based on the holy trinity of research, teaching, and service (which is often interpreted as research, research, and research). Committee members may feel they have no choice but to cast a positive vote for a known harasser. The committee should be required to consult with the students and postdocs of tenure candidates, and abusive behavior should absolutely be sufficient grounds for denial.

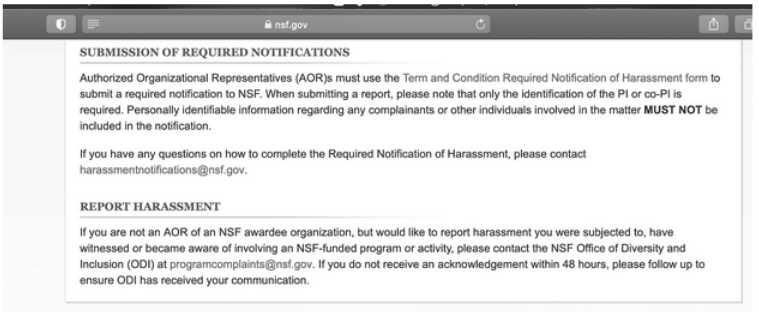

FIGURE 7. *
Screen shot taken on September 20, 2020.

As for funding agencies, if it were up to me (reminder: it is not), repeated and plausible allegations against a PI should be enough to take someone out of funding consideration until an investigation is completed. If investigations were swift and were taken seriously, then no one would be affected for more than one funding cycle. NSF funding rates are currently about 25% across the agency. Most people who apply for funding don't get it most of the time. A lot of very good science (and very good scientists) go unfunded. No one is entitled to funding, and a brief pause in eligibility to apply should not be seen as unfairly punitive. In fields where graduate students and postdocs rely on funds from a PI's grant for their livelihood, it should be possible for NSF to work with institutions to protect and secure student funding when they experience and report harassment.

Recommendation 4: Encourage anonymous reporting and transparent investigations. According to [**7**], "…NSF has developed an electronic, secure, anonymous harassment reporting system that can receive complaints directly from individuals." I have asked for such a system at every training I've attended, but if it exists that is news to me.[2] As of the writing of this article, someone visiting the webpage `http://nsf.gov/harassment` gets no information about how to report unless they are an Authorized Organizational Representative submitting a required notification to NSF. If that person thinks to click on the link for the full Term and Condition document, and if they then scroll to the very bottom of that page, they find: "If you are not an AOR of an NSF awardee organization, but would like to report harassment you were subjected to, have witnessed or became aware of involving an NSF-funded program or activity, please contact the NSF Office of Diversity and Inclusion (ODI) at `programcomplaints@nsf.gov`."

There is nothing secure, nor anonymous, about email. Worse, there is no documented process for what happens after a report is made. Does NSF conduct its own investigation? Do they simply put the complaint in a file somewhere? Is it even possible that this leads to some kind of consequences for a PI? I work at the agency and talk to the people in charge, and I honestly don't know the answers to these questions.

[2]In better news: The American Mathematical Society [**2**] has instituted truly anonymous reporting—via phone or externally-hosted web form—as part of its "Policy on a Welcoming Environment," adopted by the AMS Council in January, 2015.

Because of fear of ruined reputations and careers destroyed by false accusations (again, this doesn't happen), and mostly because of fear of litigation, institutions focus on confidentiality to the point that it actually harms their efforts to combat harassment. At NSF [**7**], reports (from individuals and from institutional representatives) are "sequestered within our Office of Diversity and Inclusion, separate from our other data systems and provided only to staff with a specified need to know." The problem, of course, is deciding who needs to know and when. Shortly after the new policy was enacted, an institution did its due diligence and reported to NSF that a PI had been banned from campus as a result of a sexual harassment investigation. The impenetrable wall between the ODI and the merit review process meant that the PI was awarded another grant after NSF had been notified.

From [**5**]: "Confidentiality and nondisclosure agreements isolate sexual harassment targets by limiting their ability to speak with others about their experiences and can serve to shield perpetrators who have harassed people repeatedly…Transparency and accountability are crucial elements of effective sexual harassment policies."

Recommendation 5: Be proactive rather than reactive about creating safe spaces. Part of NSF's efforts to address sexual harassment included a new requirement for NSF-funded conferences [**16**]: "Proposers are required to have a policy or code-of-conduct that addresses sexual harassment, other forms of harassment, and sexual assault, and that includes clear and accessible means of reporting violations of the policy or code-of-conduct. This policy or code-of-conduct must be disseminated to conference participants prior to attendance at the conference as well as made available at the conference itself."

Immediately I notice that there must be a method of reporting, but there is no requirement about acting on any reports that are filed. Do the organizers need to address anything that happens, or simply take a report and then go on with the regularly-scheduled conference activities? It's also important to note: the code-of-conduct does not need to be included in the proposal. Program Officers were told not to ask to see it. There is, right now, no requirement to include the code-of-conduct in the final report after the conference takes place. This is a completely empty requirement. When I pressed on the issue during a training, I was told that I could inform PIs about the requirement when I notified them of a conference award. As far as I know, I am the only Program Officer who does so.

For my friends who have been harassed at conferences, codes-of-conduct are essentially useless. Perhaps there is a method of reporting the behavior at the conference, and perhaps there is some action taken by the organizers or the host institution. But…what about the next conference? I know women who don't feel safe attending conferences anymore because they are never sure if their harasser will be there.

We need a process to make conferences safe (or as safe as possible) in advance. This could start, at least, with the AMS and the math institutes collaborating on an anonymous reporting system and a way to vet invited speakers and participants. No system will be perfect, but we have to be able to do better.

Recommendation 6: Stop relying on Title IX. I have already written at length about the shortcomings of Title IX offices in protecting students. The fact that NSF and other federal agencies are relying on these offices to do the investigative work, and that

departments like mine defer to them to judge if anything untoward has occurred, continues to cause problems. From [**5**]: "Judicial interpretation of Title IX and Title VII has incentivized organizations to create policies, procedures, and training on sexual harassment that focus on symbolic compliance with current law and avoiding liability, and not on preventing sexual harassment." Paula Johnson, president of Wellesley College, put it this way [**4**]: "We really have to move beyond a mind-set of legal compliance and liability and think about the ways we can change the climate."

21. Parting words

I did not want to be the voice for sexual harassment in mathematics. It's not what I set out to do. It was never my calling. A while back, I sat down to write a "service mission statement," to outline what I want to do for the profession. I realized that I couldn't do it, because there is a fundamental disconnect between what I would choose as my service mission and what has been thrust upon me.

But Dr. Córdova [**6**] challenges us to "demand accountability from those within your spheres of influence, and from yourselves." The authors of the report on sexual harassment [**5**] put it this way: "Make the entire academic community responsible for reducing and preventing sexual harassment."

So you are on notice. *You* are hereby responsible for reducing and preventing sexual harassment. I am demanding accountability from you. Get to it.

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Fiber Bundles and Intersectional Feminism

Dagan Karp

ABSTRACT. This note provides an introduction to intersectional feminism for mathematicians. It is an example of mathematical models of social structures.

22. Introduction

The goal of this work is to provide an introduction to intersectional feminism for mathematicians, and to encourage mathematicians to take action. In Section 23 we introduce language and key terminology used throughout the piece. In Section 24, we describe gender inequity in mathematics. In Section 25, we provide a brief introduction to feminism in the US, and introduce intersectionality in Section 26. We provide a brief introduction to fiber bundles in Section 27, and introduce the fiber bundle model of intersectional feminism in Section 28. Finally, in Section 29, we look to scholarship in mathematics education for guidance on actions to take moving forward.

23. Language

The moduli space of genders is neither connected nor equidimensional. It is certainly not the disjoint union of two points. It is also certainly not linear with a canonical well ordering. It is not even fixed in time. So the gender binary is an incorrect model mathematically, as is a linear gender spectrum. There is instead a rich moduli space of gender identities determined by self-identification, including people who are non-binary, agender individuals, and people with genders that are multiple or change as a function of time. The word *woman* hence of course must be used to refer to an individual who self-identifies as such at a given point it time. However historical surveys of the mathematical sciences not only assume the false gender binary, but also assign gender based on presentation. As such, there is an insufficiently analyzed disconnect between historical and current language and survey data. For an introduction to gender studies, including importantly the performative nature of gender, I refer the reader to the canonical text by Butler [**7**].

I am also intentional in my use of the phrase *intersectional feminism*. Because mathematicians use existential and universal quantifiers professionally, there is an unfortunate tendency by some in the field to propose an All Lives Matter approach to issues of equity in mathematics. While we all have multiple intersecting identities (discussed below), the focus of intersectional feminism is not, for example, people who

are left-handed and tall, or people who are geometers and algebraists. So, I use intersectional feminism in this work to highlight this movement and framework as specifically feminist, and to highlight the brilliance and work of women, especially queer women of color, who have lead the struggle for social justice and continue to do so in greater society and also in mathematics.

24. Gender Inequity in Mathematics

Gender inequity is historically severe and remains extant in mathematics. Although nearly half of all bachelor's[1] degrees[2] in the U.S. in mathematics are awarded to women according to the Conference Board of the Mathematical Sciences (CBMS) [**3**], only about 30% of the PhD's in mathematics in the U.S. are awarded to women, and further only 14% of tenured mathematics faculty are women, according to the American Mathematical Society (AMS) Annual Survey [**18**]. See Section 23 for a discussion of gender and gendered terms.

The underrepresentation of women in mathematics is persistent and pervasive. Comparing previous CMBS and AMS Annual Survey Data[3], we see that these critical transition points are persistent. For example, the percentage of women PhD recipients in mathematics has remained at roughly 30% for at least two decades. It is worth noting that such underrepresentation extends throughout the profession. For example, "women are underrepresented as authors of mathematics papers on the arχiv, even in comparison to the proportion of women who hold full-time positions in mathematics departments"[**6**]. Such patterns continue to appear in peer-reviewed publications [**33**]. An (problematized[4]) attempt was made to describe some of the landscape of obstructions to equitable participation in mathematics conferences here [**32**]; Laba's critique here is necessary reading. Women are also underrepresented in journal editorial boards in the mathematical sciences [**40**].

Gender inequity in mathematics extends well beyond representation—beyond counting bodies in the profession. The non-academic obstructions to success in mathematics for women are myriad. These include not only lack of role models and senior mentors as above, but also individual bias and structural inequities within and across institutions.

Individual bias (inherent, implicit, intentional, or otherwise) is one cause of gender inequity in mathematics. Moss-Racusin *et al* performed a seminal study demonstrating such bias in the sciences.

> In a randomized double-blind study ($n = 127$), science faculty from research-intensive universities rated the application materials of a student—who was randomly assigned either a male or female name—for a laboratory manager position. Faculty participants rated the male applicant as significantly more competent and hireable than the (identical) female applicant.[**36**]

[1] A bachelor is an unmarried man, according to Merriam-Webster, whereas a *maid* is therein defined as "an unmarried girl or woman, especially when young or a virgin."

[2] Note the persistent misogyny in the use of B.S. and B.A. degrees. Why not all maid's degrees?

[3] AMS data is archived and updated at `http://www.ams.org/profession/data/annual-survey/annual-survey`

[4] `https://ilaba.wordpress.com/2015/03/15/gender-conferences-conversations-and-confrontations/`

Similarly, in another major study, over 6,500 faculty from 89 institutions were sent emails from fictitious students inquiring about research opportunities prior to applying to a doctoral program. These emails were identical in all but the name of the students; names were randomly assigned to signal gender and race. The study found "faculty were significantly more responsive to Caucasian males than to all other categories of students, collectively, particularly in higher-paying disciplines and private institutions" [**34**]. Harvard's Project Implicit[5] has generated thousands of citations demonstrating implicit bias along axes of gender identity (among others). For example, in one publication, Zitelny *et al* find that "Implicit measures of the gender-science stereotype are often better than explicit measures in predicting relevant outcomes"[**42**]. For an important, sophisticated, and nuanced critique of the emphasis on individual implicit bias and diversity, see [**37**].

In addition to bias held by individuals, there are systemic obstructions to success for women in mathematics. Although actions are taken by individuals, when they coalesce into clearly defined patters, it is necessary to recognize bias in the underlying institution or system in place. For example, beginning in kindergarten and increasing with age, "Teachers consistently rate girls' mathematical proficiency lower than that of boys with similar achievement and learning behaviors" [**9**]. The evidence suggests that the U.S. education system is biased against women and privileges men from the very beginning of school.

Within a system of education biased against women, women face significant obstructions to success in mathematics. These include, but are not limited to, macroaggressions, microaggressions, cultural exclusion, lack of role models, lack of peer support, and sexual harassment. It is imperative to listen to women and those targeted by heteronormative misogyny themselves, as Izabella Laba persuasively points out [**29**]. I suggest looking to the blogs of Laba[6], Piper H.[7], Prescod-Weinstein[8] and the Inclusion/Exclusion[9] blog of the AMS.

A word on essentialism and reduction. I have neither the lived experience nor professional training to exhaustively analyze systemic and institutional misogyny in mathematics. Rather, here I am only trying to give a glimpse of biased systems, as motivation for engaging in feminist work. Of course it would be ridiculous and essentializing to assume that any single individual, woman or otherwise, could speak on behalf of All Women, cis and trans, regardless of income, race, ethnicity, country of origin, first language, and all other aspects of identity. As Audre Lorde points out, "It is a particular academic arrogance to assume any discussion of feminist theory without examining our many differences, and without a significant input from poor women, Black and Third World women, and lesbians"[**30**]. Also, as women are neither the architects nor the beneficiaries of misogyny, it is necessary for all people, but especially men and those who are privileged by misogyny, to engage in the lifelong process of critical self-examination and the urgent work to dismantle systems of gender bias.

[5] `https://implicit.harvard.edu/implicit/`
[6] `https://ilaba.wordpress.com/`
[7] `http://www.theliberatedmathematician.com/blog/`
[8] `https://medium.com/@chanda`
[9] `https://blogs.ams.org/inclusionexclusion/`

25. Feminism

In the Feminism and Visual Culture Reader, Amelia Jones writes

> Feminism is, of course, not a singular discourse to be easily defined or pinned down. Although its emergences (from the burgeoning of the suffragette movements in the late nineteenth and early twentieth centuries to the rise of women's lib in the 1960s and beyond) can be loosely mapped, its parameters and positions are under continual negotiation. This book takes feminism seriously but does not seek to patrol its borders by, for example, labeling authors [...] "feminist" or "not feminist." This kind of strategy is antithetical to the best impulses of what I take to be feminism. [**26**]

To give a comprehensive introduction and history of feminism is not only beyond the scope of this article, and my capabilities, but in addition I am not interested in pursuing a definitive treatment for the reasons Jones expresses above. Rather, I'll provide here a very terse introduction, intended for mathematicians who consider themselves generally unaware of the subject. There is a universe of scholarship surrounding feminist history and thought. To recommend just three texts, I suggest [**1**], [**25**], and [**38**].

Feminism (in the US) may (reductively) be described in terms of waves of the movement. First wave feminism, from the mid-nineteenth to the early twentieth century, was centrally concerned with women's legal and political rights, such as the basic rights to vote, own property, control property, and earn and control their own income (i.e. have *separate economy*). The first wave of feminism roughly ends with the ratification of the 19th Amendment in 1920. Key figures include Elizabeth Cady Stanton and Sojourner Truth.

Second wave feminism dates roughly from the 1960s through the 1980s. In this wave, feminism moved the fight from equality under the law to social equality. Second wave feminists battled sexist models of gender relations including domestic expectations for women. Key legislative victories include women's right to use birth control, have equal pay in the workplace, have educational equality, and have abortion access and reproductive freedom. Key figures include Betty Friedan and Alice Walker.

Although its roots extend back to the 1970s (or the 1670s), third wave feminism is generally described as beginning in the 1990s and is specifically *intersectional*. The gains made in first and second wave feminist movements went disproportionally to white women [**25**]. The Combahee River Collective played an essential role in the creation of intersectional feminism. Founded in 1974, they were a collective of Black feminists working to "combat the manifold and simultaneous oppressions that all women of color face"[**10**]. In the Combahee River Collective Statement, they point out that "Black women's extremely negative relationship to the American political system (a system of white male rule) has always been determined by our membership in two oppressed racial and sexual castes"[**10**].

26. Intersectionality

The Combahee River Collective point to the need to be cognizant of both race and gender in general and in movements for social justice in particular. Intersectional feminism, or intersectionality, does this and much more. The term was coined by Kimberle Crenshaw, who points out

> Intersectionality was introduced in the late 1980s as a heuristic term to focus attention on the vexed dynamics of difference and the solidarities of sameness in the context of antidiscrimination and social movement politics. It exposed how single-axis thinking undermines legal thinking, disciplinary knowledge production, and struggles for social justice.[**8**]

This then is a first and most basic understanding of intersectionality. Single-axis frameworks are insufficient. Every individual has multiple simultaneous intersecting identities, including race, gender, ethnicity, sexuality, economic class, ability status, language, and country of origin.

This basic understanding of intersectionality is crucial and has immediate implications. Crenshaw points out that discrimination against, and even violence towards Black women (and other minoritized groups) is often seen only by carefully taking an intersectional view [**11**]. This has particular ramifications for instructors. For example, it may be the case that, on a given exam, there is no performance discrepancy between men and women, nor between Black students and all students, yet it may still be the case that Black women score higher on the exam than the class as a whole. Such an observation is only revealed in an intersectional analysis.

Beyond this basic understanding of intersectionality, there is a higher level, involving intersecting systems of oppression. The simple yet extremely powerful observation is that, just as aspects of identity in individuals are intersecting and inseparable, so too are the systems of oppression, bias, and privilege governing our lives. Lorde points out that the U.S. is "a country where racism, sexism, and homophobia are inseparable"[**30**]. The Combahee River Collective not only describe themselves as being "actively committed to struggling against racial, sexual, heterosexual, and class oppression"[**10**], but, of basic and foundational importance in intersectional theory, they point out "the fact that the major systems of oppression are interlocking"[**10**]. For example, one cannot work against sexism without also working against racism. Efforts to do so discriminate against Black women, for example, and were a major impetus for Third Wave Feminism.

How can we then understand intersectional feminism? On a basic level, we must acknowledge and honor intersecting aspects of identity. On a higher level, we must also model the intersecting nature of power structures, and the way those relate to individuals. Fiber bundles are proposed as such a model.

27. Fiber Bundles

Just as it is beyond the scope of this article to provide a comprehensive history of feminism, so too is it beyond scope to give a comprehensive description of the history and uses of fiber bundles in mathematics. Instead, I'll provide a terse introduction

for mathematicians for whom this subject is new. For introductions to the subject, I recommend [**4**] and [**35**].

Fiber bundles are of basic importance in geometry and topology. Before a general definition, let's start with an example. Consider S^1 embedded as the unit circle in $\mathbb{R}^2$. Each point $p = (a, b) \in S^1$ has a tangent line at that point, denoted T_pS^1, which is the unique line through p in direction $(-b, a)$. Wouldn't it be nice if we could package these all together, and do so in a way such that

(1) the entire set of tangent lines had the structure of a topological space;
(2) points that are close in S^1 have tangent lines which are also close together;
(3) the set of all tangent lines has nice local structure; and
(4) the structure of the set of tangent lines gives geometric/topological insight into S^1?

The answer is yes, I think it would be nice and yes, we may do so. The result is the *tangent bundle* of S^1 denoted TS^1.

We now carefully construct TS^1, the tangent bundle of the circle S^1. Consider $S^1 \subset \mathbb{R}^2$. A vector in $\mathbb{R}^2$ is *tangent* to S^1 at p if it is the velocity vector of some smooth path through p in S^1. The set of all tangent vectors at p is denoted T_pS^1. Note that in this case all tangent vectors are parallel (to $(-b, a)$), and we recover the tangent line at p, but this construction works much more generally. Then the *tangent bundle* is defined by

$$TS^1 = \{(p, v) \in S^1 \times \mathbb{R}^2 \mid p \in S^1,\ v \in T_pS^1\}.$$

We leave it as an exercise for the reader to show that TS^1 is diffeomorphic to the infinite cylinder, $TS^1 \cong S^1 \times \mathbb{R}$. See Figure 8.

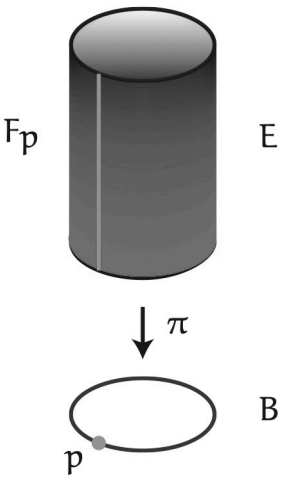

FIGURE 8. The Tangent Bundle of S^1

With this example in mind, let's now define a fiber bundle in general[10], following [**4**]. Let G be a topological group which acts effectively on a topological space F

[10]There are many equivalent definitions. A good way to get to know a geometer is to ask how they think about fiber bundles.

on the left. A surjective map $\pi : E \to B$ between topological spaces is called a *fiber bundle* with fiber F and structure group G if B has an open cover $\{U_\alpha\}$ together with fiber-preserving diffeomorphisms

$$\phi_\alpha : E|_{U_\alpha} \cong U_\alpha \times F$$

such that the *transition functions* $g_{\alpha\beta}(x) := \phi_\alpha \circ \phi_\beta^{-1}$ are continuous functions with values in G.

The tangent bundle of TS^1 is a fiber bundle with fiber $\mathbb{R}$ and structure group $GL_1(\mathbb{R}) \cong \mathbb{R}^*$, the multiplicative group if units in $\mathbb{R}$. Indeed, $\pi : TS^1 \to S^1$ is simply the projection map. Let $\epsilon > 0$ and define an open cover $\{U_1, U_2, U_3\}$ by

$$\begin{aligned} U_1 &= \{e^{i\theta} \mid -\epsilon \le \theta \le 2\pi/3 + \epsilon\} \\ U_2 &= \{e^{i\theta} \mid 2\pi/3 - \epsilon \le \theta \le 4\pi/3 + \epsilon\} \\ U_3 &= \{e^{i\theta} \mid 4\pi/3 - \epsilon \le \theta \le \epsilon\}. \end{aligned}$$

Then the U_i intersect pairwise and the transition functions are indeed invertible linear maps on $\mathbb{R}$. See Figure 9 below.

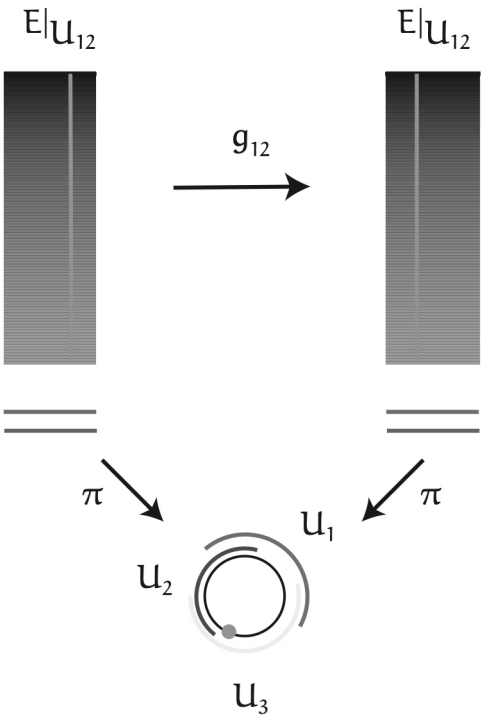

FIGURE 9. Transition Functions and Local Trivializations in TS^1

28. Intersectional Feminism as a Fiber Bundle

The fiber bundle model of intersectional feminism is quite simple. The base space is society, the set of people (in whatever society we are theorizing). An open cover consists of aspects of identity and their refinements. This open cover of the base space reveals the basic and first observation of intersectionality, that people have intersecting and inseparable aspects of identity, such as race, gender, and sexuality: we each occupy the intersection of multiple sets. But above each of us lie the preimages of these open sets; above us lies a complex web of social structures and systems of oppression, such as racism, misogyny, homophobia and transphobia. These preimages above, the local trivializations, reveal the higher structure of intersectionality, that systems of oppression are also intersecting and cannot be separated. The transition functions preserve the structure of the bundle, illuminating the intersectional premise that the social fiber above each individual has the same structure and is preserved as we operate within biased systems.

And what is the fiber? Building off bell hooks [**24**], the social fiber in the U.S. is ableist cis-heteronormative white-supremacist capitalist patriarchy. Just as points in the base space of a fiber bundle all have isomorphic fibers above, so too do all people in society live under systems of oppression. (Of course the impact of these systems differs across people; that's somehow the point.) Just as transition functions preserve the structure of the fiber, so too in our interactions between people are we living under the influence of our biased social fiber. This is consistent with the hegemonic nature of oppression.

29. Praxis

As the saying goes, in theory there is theory and practice, but in practice there is only practice.[11] What practices work toward a society that is no longer accurately modeled as a bundle with ableist cis-heteronormative white-supremacist capitalist patriarchy as fibers? In particular, how may we work against oppression in and through mathematics? Fortunately there is a wealth of scholarship and lessons learned from past practice of which we mathematicians may avail ourselves.

Within mathematics education, intersectional feminism plays an important role in critical pedagogy theory and practice. In her groundbreaking text Teaching to Transgress, bell hooks brings an intersectional feminist lense (and much more) to critical education theory [**23**]. Critical education theory goes back to Paulo Freire; it challenges the banking model of education and posits education as a liberatory practice [**13**, **16**]. Through engaging with Teaching to Transgress, one may adopt intersectional feminist practices in education. Scholars including Frankenstein [**15**], Gutstein [**20**], and Tutak [**41**] bring critical pedagogy specifically into mathematics education.

This opens the door to the world of (mathematics) education informed by and consistent with intersectional feminist practice, including but certainly not limited to anti-oppressive education [**28**], social justice oriented (mathematics) education [**21**, **27**], critical race theory in mathematics [**14**, **31**], and ethnomathematics [**12**].

[11] To quote Benjamin Brewster in the Yale Literary Magazine in 1882, "What does his lucid explanation amount to but this, that in theory there is no difference between theory and practice, while in practice there is?[**5**, p.202]"

To highlight in more detail the work of a few individuals, Estela Bensimon has research-based suggestions for individual and institutional self-analysis and improvement. Following her Diversity Scorecard, individual instructors may analyze their grades by race and gender [**2**]. Focusing on an intersectional lense, we should pay attention to these data not only separately, but also in the intersection, lest bias against low-income students with disabilities, or women of color, go unreported. Similarly we should take an intersectional feminist lense in all our classroom practices, including not only our gradebooks, but our co-construction of classroom dynamics, our journey toward equity centered pedagogy, our efforts toward rehumanization, and universal design.

At the institutional level, we must also use intersectional analyses. For example, Bensimon's Diversity Scorecard is not only a tool designed for instructors, but rather it is a method of institutional analysis [**2**]. We should push our institutions to perform such institutional intersectional analysis, for example as overseen by the University of Southern California's Center for Urban Education, in the case of the Diversity Scorecard. Similarly we may turn our attention to hiring committees, tenure and promotion committees, departmental and institutional leadership. Such structural changes have potential for significant impact.

Turning to another scholar, Nicole Joseph has a specifically intersectional lense in mathematics education research. She explores the intersectional experiences of Black women [**39**]. And she analyzes ways to disrupt practices that minoritize Black women in mathematics using key frameworks from critical race theory [**22**]. Mathematicians interested in finding research-based practices working toward intersectional feminism in mathematics education need only look to her oeuvre.

Finally, Rochelle Gutierrez pushes past equity toward rehumanization [**17**], and beyond rehumanization to the world of Mathematx [**19**]. Her work is very broad, and includes intersectionality foundationally. Mathematx is conceptualization of the practice of mathematics which incorporates ideas from many sources including postcolonial theory and indigenous knowledges. Just as I generally don't recommend scheme theory to first-year undergraduate students, I don't recommend Mathematx for people unfamiliar with ethnomathematics, critical race theory, gender studies, and some grounding in mathematics education. But it is worthy of the prerequisite study; it is powerful, beautiful, important and imperative.

The presence of systems in mathematics which bias some and privilege others dispels the myth of the meritocracy. We must continuously work to dismantle systems of oppression, to change the fiber of our profession, for the benefit of the mathematics community and beyond.

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On Parameters for Communicating Mathematics

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ABSTRACT. Mathematical ideas can be communicated in many different ways. As for teaching, especially in service courses, it is important as a teacher to be able to adapt the language, the notation, the abstraction level and the difficulty level to an ever changing and increasingly diverse student body. Having folks in the faculty who have the time, the sensitivity and the training to deal with these difficulties is more and more appreciated. Being aware of such issues can be especially challenging in the field of pure mathematics. I want to share here from my own experiences as a mathematics faculty who specializes more in teaching and support rather than in research. There will also be some comments on the role of non-ladder faculty in colleges, positions which come with some unexpected advantages. An example is the absence of publication pressure. The freedom to contribute to any particular area of mathematics allows to branch out into other fields, and explore lighter mathematical topics which are closer to topics taught in service courses. This is illustrated with an example where some new mathematics was created in this environment.

30. Introduction

30.1. In my experience, teaching is extremely rewarding and fulfilling if it works. Since it also happens in a surrounding, where one does not have control about everything, there can be quite a bit of drama in pedagogy. Teaching can be especially adventurous for creative teachers who like to experiment and try out new things or question existing practices or even try themselves in edutainment [**7**]. Pedagogy is also a subject, where theory and experience do not always line up. Similarly to a beautiful physical theory which can be "lost in math" [**37**], a paradigm for learning might not work for everybody as intended. The experiment is the actual classroom. There are many factors which contribute in the teaching process. Some aspects of this adventure are illustrated in [**27**]. The workshop on "Professional Norms in Mathematics" at Johns Hopkins illustrated in various ways how some parameters are difficult to control as they depend on overall norms in society, on expectations from the workplace, on funding situations, as well as on existing education structures in K-12 education.

30.2. Many of us mathematicians would of course like to axiomatize the teaching process, have theorems stating truths which are eternal and which can be adopted and mastered and which, if followed, should lead to successful teaching. But this goal can hardly ever be achieved. Teaching deals with many different humans embedded in

Notes to a talk given on September 21, 2019 at Johns Hopkins.

a society subject to constraints, perception, opinion, social norms, fashion, anxieties, challenges and feelings. In my experience, a major reason why theory and practice in pedagogy do not always align is that models about student learning often do not take into account the teacher and the surroundings in which the model is applied. In particular, the personality of the teacher, the teaching culture at an institution, the classroom size, the student expectations and the preparation levels of the students can be very different from person to person or from place to place. And then there are mechanisms which are hard to control. Psychological and bias issues, sometimes entrenched over centuries, can surface. This happens especially in highly interactive classrooms. Such parts are often hard to predict and even harder to manage.

30.3. But it is far from hopeless. While teaching, there are some parameters which I'm able to control relatively easily without having to change society or previously entrenched education practices. I would like to focus on one or two such parameters. There is a particular important yard stick, which has been the hardest to control: it is the choice of the **abstraction level** in communicating mathematics. While abstraction can be hard to gauge, it can be managed with training, through experience and especially through experimentation and by getting lots of feedback. It also has a chance of being quantified. Maybe not as much as the parameters "complexity" and "difficulty" but it is easier to assess than "beauty" or "relevance".

30.4. One of the main points I want to make in this article is that "abstraction" is a crucial ingredient to be aware of when teaching mathematics. I also aim to illustrate why this parameter is hard to gauge. I have seen mathematicians with impeccable personal sensitivity, taste and skill that have got it wrong, as can be seen especially when observing talks in research seminars or conferences. I myself often get it wrong. The issue surfaces everywhere, even when I had been preparing for the current talk. I also want to illustrate it with some new mathematics later on. It will be up to the reader to judge whether I misjudged the abstraction level in the example I give.

30.5. Here is an other non-mathematical illustration: until shortly before the meeting, I had planned to use the talk title "abstraction diversity". It sounded good because this word addresses a core message of this talk: be aware of the abstraction spectrum and how to cultivate its variety. But when floating this to a colleague, she asked "what does abstraction diversity actually mean" and indeed, if one has not been told the story, the word does not say much. It does not work as a title because it is itself too abstract. The episode illustrates that "abstraction" comes in essential in any taxonomy about communication, even if it is not mathematical.

30.6. The subject of "parameters" allows me also to share a few general pedagogical remarks related to taxonomies. Taxonomies are practical tools which have proven to be useful. This is illustrated by the vastly popular **Bloom taxonomy** [**4**, **10**], which when simplified splits things into **Factual**, **Conceptual**, **Procedural** and **Meta-cognitive** parts. In [**86**] the Bloom approach is called the "Swiss Army Knife of Curriculum Research". I myself learned about such taxonomies during a workshop organized by the Harvard Bok Center and later while directing a thesis in the "math in teaching program" in which taxonomies, meaning systematic categorizations of educational mathematical areas, were the central part.

30.7. Taxonomies in education are what coordinates are in geometry. One can use them to quantify and visualize teaching parameters. A mathematical lecture, a talk, a mathematical problem, a mathematical theory, a speech or a paper or book can be placed into such a parameter space. For me, as a teacher, such yard sticks have been extremely helpful, especially for lesson planning and in order to find the right approach for a specific audience. It is a trusty guide when preparing a presentation, a talk or a class. From personal interactions with teachers and just by looking on the shear amount of material which is available online, one can extrapolate that many other teachers feel similarly about this guidance tool.

30.8. One of the things which attracted me to teaching as an undergraduate student, and prompted me to become a course-assistant in calculus and computer science early on, is that there are so many different approaches which can work. Already in high school as well as college, I liked to observe and analyze the teaching styles of my teachers. I was fortunate to witness a wide spectrum of techniques which illustrated that brilliant teaching can be done in many different ways. Some classes were interactive, some less, some were improvised, some choreographed carefully, some would try new paths while others would follow more traditional ways, some would use props or make interactive experiments, even games played with the class, others would just present pure knowledge and focus on clarity. It is important to me that we should not only embrace diversity in society but also in classroom cultures.

30.9. In the third and last part of these talk notes, I would like to pass along some general words of wisdom or observations I have collected over the last 30 years, while learning mathematics and teaching, both as graduate student and as a non-ladder teaching faculty. I'm aware that the act of giving advice runs the risk of being perceived as condescending or even arrogant, but I believe that we have to honor not only the advice of highly successful people but also have to listen to ordinary folks like me who work in ordinary normal circumstances. In any case, I believe that sharing experiences is important and that the cross fertilization of (sometimes contradicting) ideas was one of the many wonderful parts of this workshop. It goes without saying that much what follows is based on personal experience and not on mathematics education.

30.10. Preparing this talk allowed me to start aligning some thought snippets I kept collecting over years and organize them, maybe to summarize in a book, once retired. Many of them concern with the role of the "**invisible university**". [**18**] The original name "invisible university" relates to the complex topic of the profession of teaching educators [**70**] or teaching faculty in community colleges [**49**]. At the moment, there is no time for that yet, as I try to use my free time to pursue some mathematics research on my own. In my position, research has become a hobby but it remains an activity which builds strength for teaching and allows for relaxation. Every semester brings new challenges, new experiments and new students.

31. Acknowledgments

31.1. I would like to thank Emily Riehl for organizing this unique, rich and inspiring conference in Baltimore and for the kind invitation to participate. I consider myself a rather exotic creature in the landscape of pedagogy and certainly am a bit of an outsider, so that the invitation came as a surprise, but I believe it is fitting in a time

of increased recognition of diversity to listen also to a mostly invisible and quieter part of the teaching landscape. Yes, we are mostly invisible, but we are also proud of what we do and are also grateful to be able to work with undergraduate students, graduate students as well as other faculty and staff.

31.2. To add while revising this in October 2020, I also want to express sincere thanks to over hundred of constructive improvements suggestions contributed by referees who were reading the text. One of the main referee concerns had been that some of my statements had appeared without references to math education research. I tried to address this now locally, where relevant and labeled it clearly as opinion, where needed. It should again be stressed that this document is far from an education research paper. It contains sometimes rather personal opinions which are not backed up by data. I'm immensely grateful for the referees to identify these spots, make many corrections or point out places where things need to be clarified. This also required to expanded the bibliography at some points.

31.3. My perception of pedagogy is in constant flux. This is also a reason for why the subject is so attractive to me. Regular reflection about teaching makes the process also more rewarding and interesting to me. My perspectives are influenced from student feedback, other teachers and projects. Some of the best insights do not come from surveys or feedback forms but from actually teaching in the trenches or even from grading assignments and exams. [Added in proof in October 2020, it must be said that the temporary move to online teaching has already started to produce even larger shifts in my own perception of teaching and already produced more insight. It will certainly be important to assess and reflect on all this again in a few years.]

31.4. I benefited a lot from undergraduate students and teachers in the "math for teaching program" as they reached also in pedagogical areas. While teaching mathematics to an "artificial intelligence bot" in 2004, we gained some insight about teaching aspects [**43**]. The area of teaching machines is especially accessible because psychological parameters are absent. We do not yet have to inspire or motivate them. A thesis of Elizabeth Slavkovsky explored in 2013 the feasibility of 3D printing in the classroom [**45**, **74**] and the work with Jose Luis Ramirez Herran on omni-vision in 2009 (i.e. [**44**]). Such projects were not only interesting for me in the context of pedagogy but also got to an exciting interplay of newer technology with teaching. It also changed the way I think about mathematical objects and how to present them. A project of Paul Hermany "Math puzzles, Games and Activities" from the Fall 2016 got me more exposed to taxonomies, a topic we will start with in the next section. Allen Lai (a Harvard undergraduate) worked with me for one semester on Gamification in Mathematics Education. It is a rather unexplored area with a lot of potential. This is an exciting world. Another angle came in from a project of Ethan Fenn, "The Uses of Spurious Proofs in Teaching Mathematics", a thesis written in the Fall 2017. It studies the use of paradoxes and fallacies to explain a topic also with the help of taxonomies. I mention these examples also to illustrate that teaching goes both ways.

32. Taxonomies

32.1. When thinking about parameters, **taxonomies** can be a useful tool. For a mathematician, these are parametrizations of teaching areas. They build coordinate

systems in larger dimensional spaces of pedagogical parameters. To illustrate this, let me mention a handy tool which is useful when looking at any teaching communication. A topic for a class, a talk, project or text can be quantified in the *ADC* **parameter space**, where *A* stands for **abstraction** level, *D* stands for **difficulty** and *C* stands for **complexity**. There are only three parameters on purpose. It is simple to remember and can be applied without getting lost in too much meta-complexity.

32.2. The **complexity** level of a mathematical task can be quantified quite easily. It essentially measures on how long one has to work on the problem, if one knows how to do it. Computing the product of two 20-digit numbers is a complex task, it is not difficult and not abstract. It is just "tedious". Also search problems which require to try out many cases can require a long time. An example is the problem to place 10 non-interacting super queens (figures which can move both like queens or knights in chess) configurations on a 10×10 board. Up to symmetry, this problem has a unique solution.

32.3. The second parameter, the **difficulty level** of a problem can be measured by how long one has to work until one knows how to start. We sometimes say that a problem is "messy" if the complexity level is hard and "hard" or "tricky" if the difficulty level is high. These are different things. Deciding whether a Diophantine equation has a solution or not can be hard and the story of "Fermat's last theorem" shows this. An example of a doable but still hard problem is to prove that $(x, y) = (3, 5), (x, y) = (3, -5)$ are the only integer solutions on the Mordell curve $y^2 = x^3 - 2$. The problem was solved already by Fermat (he at least claimed to be able to prove it) but solving it needs an idea. When given the problem, one is stuck at first. There is no obvious way to start solving it. For the history of Mordell curve see [**58**]. For more about the above Mordell curve, see [**16**].

32.4. I have found that the third parameter, the **abstraction level** is a quantity which is the hardest to catch or even to estimate. This is the case not only when teaching a mathematics course, it is also the case when writing a mathematical article, when designing a program or when presenting a topic in a seminar. From my experience, the general rule is that one always over estimates the ability of the audience or user. Abstraction is also subjective because it very much depends on the training or exposure to such thoughts. An example: we all are familiar on how to go from integers to rational numbers by looking at pairs x/y of numbers (even so it can be tough topic to teach [**8**]). Fractions have been developed early by Babylonian or Egyptian mathematics [**31**]. There is an abstract version of this when doing a Grothendieck completion of a monoid to a group [**6**]. This construction by itself is neither difficult nor complex, it is the same idea as constructing fractions, but the more abstract and more general framework makes it less accessible at first.

32.5. If abstraction is so difficult to quantify, why do we like abstraction at all? One reason is that it leads to "elegance", "rigor" and "brevity". Calculus was developed first rather intuitively but errors in judgment required to go deeper. Both the process of building a foundation as well as the process of building rigor forced mathematicians to build in more abstraction. In calculus, abstraction increased especially during the 18th century and the abstraction has since increased even in an accelerated way [**33**]. There had been push-backs and the "new math" [**41**] or the "math wars" controversies [**48**]

illustrate this. Abstraction is not bad in and of itself - the Grothendieck completion for example is extremely powerful in mathematics - but it needs training to master it. I found that teaching abstract material requires more skill and a feel of what a student is ready for.

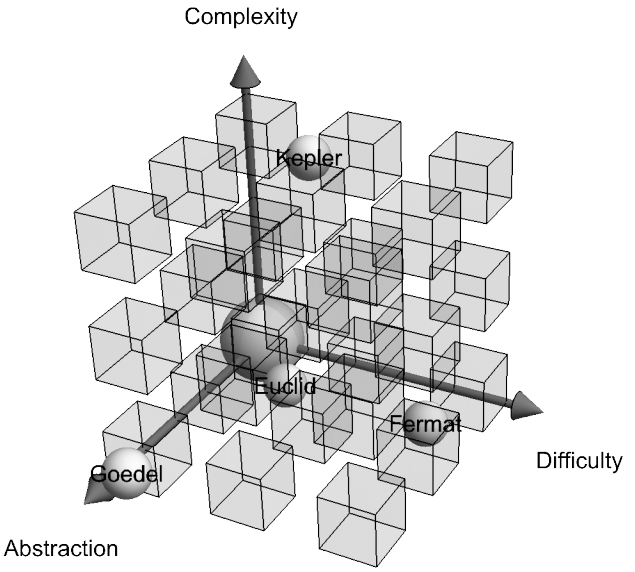

FIGURE 10. The abstraction-difficulty-complexity space is a simple taxonomy to label a mathematical problem. The simplicity of this scheme has purpose: taxonomies which are too complicated can not be observed and used well any more. While difficulty and complexity are relatively easy to measure for a teacher, I observe that the abstraction level is often difficult to gauge and often misjudged.

32.6. In my experience, taxonomies in general can help when preparing to teach. Of course, there are many other parameters which can matter when placing a mathematical problem in the landscape of all problems. The **amount of applicability** for example is an important one. In general however, I have found that the feasibility of applications is often misjudged badly, even by expert teachers. For example, using an application from physics, biology or sports requires background knowledge in that field. Even if everything is explained, the student has to be exposed to the topic already. This can be cultural: I can relate to soccer for example much more than to baseball because I grew up in Switzerland, where soccer is popular. Things can go worse then if more and more details are added for clarification: explaining one thing requires to introduce in general three more things, leading to an explosion of explanations which can bury the underlying math problem.

32.7. Relatively new is the urgent push of the use of "data" which add an other dimension to taxonomies. Our calculus courses for example were now required to satisfy a quantitative reasoning with data requirement (QRD). The committee approving courses requires that mathematics courses have to include problems with larger data sets. It was explicit in the recommendation that one has to deal with explicit messy data sets. I view this difficult because Mathematics is about elegance. It is poetry and beauty [**2**, **35**, **57**, **69**, **73**, **84**, **85**]. Still, the benefits of the QRD requirement were great. The discussions forced us to think about fundamental questions and also to reevaluate which topics need to be taught. Data science and calculus seem have little in common at first, but there are many interesting connections.

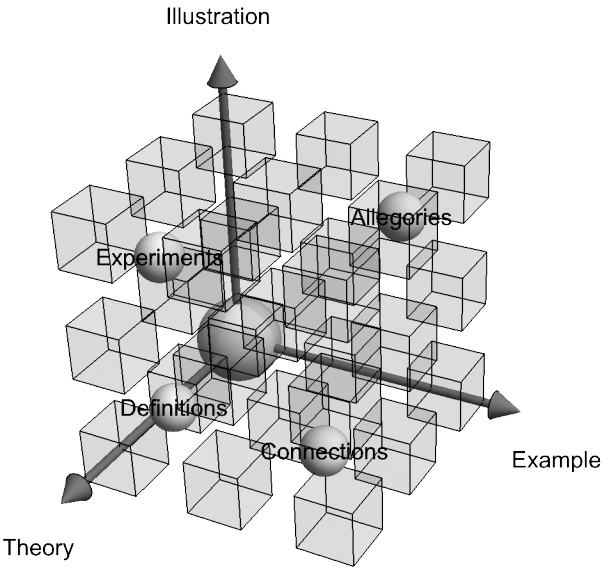

FIGURE 11. The **Theory, Example, Illustration** space is a three variable taxonomy to organize lessons. I feel that keeping it simple is key. We can not keep track of too many things when preparing for a lecture. But knowing about these parameters helps to keep a balance. A lecture, a book, a presentation needs all three ingredients.

32.8. Another interesting parameter space is a taxonomy which encodes **"Knowledge, Algorithms and Concepts"**. We realized when teaching calculus to a bot [**43**], (which is now done on a large scale by essentially all AI related companies), that these are three fundamental parts of teaching. We called it there the "What-How-Why" parameter space. The idea is that we have to first transmit definitions and language and terminology, then learn procedures and algorithms and finally plant insight and understanding. The first two parts usually pose little difficulty to teach, the last one is

difficult. The Bloom taxonomy is very close but contains more details. Bloom merges in also an assessment part. We actually thought of doing an assessment in each of the knowledge, algorithms and concepts parts separately.

32.9. Another simple taxonomy is the **"Theory-Example-Illustration"** space. I have often observed that good examples and pictures are essential to understanding a theory; but examples alone or illustrations alone do not work without theory. Examples are important to explain an abstract topic. Conversely, I have found that theory in the form "Theorem-Proof-Example" can be as off-putting as using examples only. Illustrations have pitfalls too. There is the problem to illustrating the obvious things well but then brush over more complicated things. The example in the next section can illustrate this. The reader can then judge whether the mix of theory, example and illustration work or not.

33. Illustration

33.1. I brought models of the 6 positive curvature 2-graphs to the conference and presented part of the "Mickey Mouse sphere theorem" that was at that time being written down. The theorem says that every positive curvature d-graph is a d-sphere. It captures an aspect of of heavy sphere theorems [**9**, **20**] in differential geometry: in the discrete setting, things are simpler. One reason is that the strong positive curvature assumption prevents projective spaces. The octahedron is a constant curvature 2-sphere with 6 vertices of curvature 1/3 and the icosahedron is a constant curvature 2-sphere with 12 vertices of curvature 1/6.

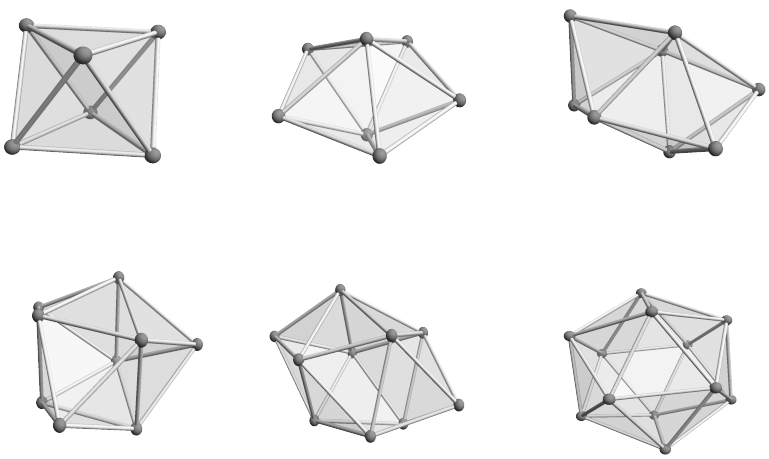

FIGURE 12. There are exactly 6 positive curvature 2-graphs. They are all 2-spheres.

33.2. I liked to mention this result in the "professional norms" set-up because I found that there is often little appreciation for simple things in math. Elementary topics are in danger to be dismissed because they are "not deep enough". It is the job position I find myself in which allows to do such things off the chart, away from mainstream and also ignore general perception and more importantly, general trends, opinion or fashion. The theorem also allowed me to demonstrate first-hand some teaching techniques, like bringing models to the classroom or to have students compute and count with such models in small groups during the class time. During the conference, the "class" was successfully computing the Euler characteristic of all 6 graphs by summing up the curvatures at the vertices. I like the Mickey Mouse world. The term "Mickey Mouse mathematics" appeared in [**76**].

33.3. In order to state the general Mickey Mouse sphere theorem in the higher dimensional case, one has to use clear definitions of what a discrete manifold and what a discrete sphere is. While one can illustrate the two-dimensional case easier, I try here to explain the general case in arbitrary dimensions. The proof is not difficult and also in higher dimensions still uses an argument we know when playing with magnetic sticks and balls. A given embedded wheel graph can be extended to an immersed two-dimensional surface, like when we are are knitting a surface. (By the way, I loved knitting as a kid. It is a beautiful mathematical process to constructing complex topological shapes from simple yarn. I believe that it was also one of the reasons to fall in love with geometry.) Once we have extended the surface and closed it, it has to be one of the 6 possible positive curvature 2-spheres. This shows that diameter is maximally 3, in arbitrary dimension. One can then cover the graph with two patches which are balls and conclude that we have a sphere.

33.4. In order to formulate the Mickey mouse sphere theorem, we use recursive definitions and use the notion of unit spheres $S(x)$ in a graph, which is the graph generated by all vertices directly connected to x. The empty graph 0 is declared to be the (-1)-sphere. The 1-point graph 1 is declared to be contractible. A d-graph is now a finite simple graph G for which every unit sphere is a $(d-1)$-sphere. A d-sphere is a d-graph G for which there exists a vertex x such that $G-x$, the graph in which x and connections to x are removed, is contractible. A graph G is declared to be contractible if there is a vertex x such that $S(x)$ and $G-x$ are both contractible. A graph is defined to have positive curvature if every wheel graph that is embedded in G has either five or six vertices.

33.5. The theorem is:

Every connected d-graph of positive curvature is a d-sphere.

33.6. There are exactly 6 positive curvature 2-graphs. It is a still unexplored question how many positive curvature d-graphs there are in dimension d. As the diameter of the graph is 2 or 3, there are only finitely many. There are positive curvature graphs in any dimension. One example is the d-polytope, which is obtained by making discrete suspensions (double cone extensions) over the $(d-1)$-cross polytope. The 1-cross-polytope is the cyclic graph with 4 vertices, the 2-cross-polytope is the octahedron with 6 vertices, the 3-cross polytope has 8 vertices etc.

33.7. As a meta remark, we want to encourage the reader to observe how I have used here the concepts of "theory", "example" and "illustration" to present a new topic. The illustrations have been placed in this document not just by chance but with the goal that they can serve a purpose. A picture might take away quite a bit of space but a picture can say more than 1000 words. On the other hand, when presenting theory, it can be important to be as clear as possible. Examples help to get intuition. I hope having been able to achieve this goal in the short exposition above. If not, then I again have misjudged the abstraction level of the presented material on the Mickey Mouse theorem so that I have to refer to [**42**] for more details.

34. Some pedagogical remarks

34.1. The following remarks are personal observations. They are not descriptions of analysis or reflections on studies in mathematics education. Indeed, they often tap into areas, where obtaining quantitative data would be rather difficult. The personal experiences of teachers can be different and depend on many parameters, especially if it concerns personalities.

34.2. Easier mathematical topics are not necessarily easier to teach.

At first sight, it appears that more elementary topics are easier to teach than advanced topics. The main reason for the fallacy is that the audience has to be factored in. Beginner topics are taught to less experienced target subjects and are in general also absorbed by a larger and more diverse population which has not yet chosen science as a calling. I myself have taught mostly to undergraduate students who need mathematics as a prerequisite for their degree. It is quite common however that students taking such service courses then get the taste for more advanced mathematics or even concentrate in a mathematical field. I myself taught as a substitute teacher in high-school for a couple of weeks, taught undergraduate to graduate courses in colleges. In retrospect, the high-school teaching was for me the most challenging to teach and the graduate level courses with graduate students in the classroom were the easiest to teach (even so the preparation for the later could be enormous). I myself like the challenge of explaining simpler things as well as observe mathematical grand masters doing this like [**25**, **72**] teaching elementary algebra topics.

34.3. Also, after more than 30 years of teaching in colleges, teaching service courses remains for me a challenge and therefore has remained a rewarding activity. There are various reasons why things do not necessarily get easier with time: with more and more teaching experience, one has also to select and chose topics which are adequate and relevant and again and again adjust the correct and adequate level of abstraction, the difficulty and the complexity. One has also to invent new approaches if one wants as a teacher to remain excited about the topics one is teaching. For the later, new technologies like for example advances in web technologies, in 3D printing, in computer games, in computer algebra systems or computer graphics have been helpful to me.

34.4. And then there are parameters over which the teacher has less control: there is an ever changing level of student maturity, there is competition of other departments and other courses, there are changes in prerequisites as well as more diversity in learning. As more advanced a topic is also, as more mature and selected the student body is

also. I noticed that quite many students on a higher level math course have acquired learning techniques which allow them to perform error correction or overcome bias assumptions. Especially for experts it can be difficult to hit the adequate difficulty level when teaching a subject. I have witnessed this when watching talks at conferences or seminars and myself spent a lot of time and effort to bring a topic to the right level.

34.5. We have to deal with ever changing **student preparation levels.** In a fast changing world, these changes can happen within a few years.

34.6. The level of preparation and maturity of a student are things over which we often have little control about when teaching a course. Yes, there are placement tests which filter students, but at most schools, they only serve as a recommendation which the student can override, often even against recommendations in personal advising sessions. It is the task of the teacher to read the preparation level of the class and adapt the course if necessary. This can be challenging if a specific goal needs to be reached. Science progresses fast. Just teaching less can not be done without good judgment about topic selection.

34.7. I noticed that there are college-relevant mathematical skills which could be taken for granted 10 or 20 years ago which are less present today. The PISA results [**63**] indicate only a slight decline of mathematics abilities in the US from 2003 to 2018 but an increase in science score points. These data do not back up my claim about having less college relevant math knowledge but they also test different things. In mathematics what can matter a lot are **geometric intuition** as well as **algebra mastery**. It would be wrong to blame the high schools, parents or the students for a lack of preparation. I speculate that a major reason is a diversification of knowledge which happened through rapid changes in information technology as well as in the culture of teaching media [**54**, **71**]. Students might already have to learn to write computer programs early on, or then are exposed to statistic classes earlier. They might take a computer aided design course or a robotics course.

34.8. Students often struggle with fundamental mathematical techniques. Therefore, **mastering basic techniques** remains important.

34.9. I observed that in the last 20 years, basic algebra mastery for college students has dropped. I personally explain this to the tendency of modern education to focus on conceptional understanding and neglect also to include some drill. Looking at conceptional understanding is a good thing, but one has also not to forget that a solid mastery of basic algebra techniques frees up computing space in the brain for larger scale thinking. What happens is that a student with less practice has to spend precious thinking time on doing arithmetic, a more experience writer can do the computation "automatic" and in the same time think about also "why" this is done, make connections to other fields or reflect on whether the result "makes sense" or "explore what goes beyond". The reason is that can free up time to be more creative. Practice makes perfect in almost any human activity.

34.10. To put it more bluntly or even to provoke: spending some time with "drill" can pay off exponentially years later. More advanced topics are absorbed faster, and higher levels are reached faster later on. A student who has to look up how to differentiate a basic trigonometric function or can not simplify equations is swimming in oil, gets tired and can not enjoy the progress. Such a student can drown in a mess of errors. Therefore, I feel that some time has to be spent with skill training. Repetition is necessary in sports or music or when learning a language or programming. It remains also a healthy habit in mathematics. The lack of basic algebra mastery appears to me as a major stumbling block to get ready for more advanced courses. [As expected, I was asked by referees to back up such a claim up by research in education or psychology. The only thing I can do here is to draw from experience and observing thousands of students personally and watching them doing mathematics. I also can draw on my own experience when learning mathematics, music, languages or sports or games. Activities start to become more fun for me when I'm better in it, and often getting better includes repetion. Practicing "finger etudes" when playing piano, learning vocabulary for languages, pushing weights in sports or do things over and over again when playing games so that reflexes kick in.]

34.11. It appears to be a challenge: how can one make repetitive exercises more fun and less routine? There are various approaches. One has tried for example to wrap it into games [**38**, **51**]. I'm convinced that many parents who has had kids in the last 20-30 years must have tried this. One problem with games is that it is difficult to find the right level. I myself have sometimes difficulty with mastering a computer game. The reason is also lack of exercise. It sometimes requires that one has to spend enormous time to practice in order to be able to finish a game or that one gets drawn into parts of the game which have little educational benefit. As the gaming industry still grows and has moved into educational domains, we will learn more about this in the future. I myself advocate not to overthink it. The fun with practice can if the repetition becomes "meditation". Almost always when doing something over and over again, new variations, new patterns and even new insights appear. I myself experience that whether it is in music, in math or running long distance, there is something relaxing about repetition.

34.12. Sometimes grinding through an exercise and variations leads to a new idea, like in science, where break-through ideas often come only after trying out many wrong paths [**64**]. We would never dare to question that basic repetitive experiments in biology or chemistry are necessary to make a break-through. Modern students are often told that it is "not cool to memorize", that repetition can be done by a computer. While there is some truth in it, I think that this feels nihilistic. With the same logic, we could say that we do not have to exercise any more as we move around with cars, we do not need to play chess any more because computers win against us or we do not have to read any more because a bot can do that better, we do not need to learn a language any more because there apps which translate, we do not need to play a music instrument, because synthetic music has started to replace musicians.

34.13. Even in a time when an entire library fits in a smart phone, it is still important to know. **Knowledge helps to build connections**.

34.14. Since all information is easily accessible even from a smart watch, it can lessen the motivation to **know things**. Intelligent assistants like "Siri", "Alexa" or "Google" know how to compute and know many definitions of mathematics. Why do we need to learn any more? One reason can be that we live in a time, where knowledge is accumulated at a tremendous pace. As we discover more and more mathematics, it also becomes more and more important that we know some landmarks in that large and ever growing landscape. It allows us to build new connections and eventually to become more creative.

34.15. Being discouraged to learn and master techniques because of new technology is not a new phenomenon. When calculators appeared in school, I felt a decreased motivation to do arithmetic in the head or on paper. With computer algebra systems, it decreased my motivation to do complicated integrals by hand. This has led to a decline in the ability to do computations, as I can notice that myself. With the emergence of smarter computer algebra systems, I can develop and test most classroom examples also computer assisted and despite teaching calculus for a living, would have to practice to integrate more complicated quotients of trigonometric polynomials. While the help from computers has also freed up time to do more interesting things, true insight for me still comes from actually doing the process also by hand, at least in simpler examples.

34.16. The emergence of the now omnipresent internet makes it less likely that a student understands the need for a rudimentary encyclopedic knowledge. It does not make sense to lament about this. It is a fact that we can not only do computations with a computer, but also look-up knowledge very quickly these days. While this is great, still, knowing how to do an algorithm like a polynomial division or do do square root computation on paper has its benefits. Knowing basic things like the quadratic formula is a prototype 'knowledge pillar' that can help to foster interest in more advanced topics like Galois theory and also appreciate historical connections, like when algebra was done more geometrically and completing a square had geometric meaning. It is important to "feel comfortable". Knowing how to do things, sometimes makes one feel comfortable. The example of the Rubik cube [**39**] with speed cubing competitions illustrates that there can be also a lot of joy in mastering an algorithm fast. These competitions are still very popular.

34.17. In order to understand a topic well, one has to understand it also **well beyond the level** at which it is taught.

34.18. This is certainly my own opinion. The somehow idealistic statement can be challenged. The subject to teach can be so advanced for example that already mastering the minimum can be overwhelming. I personally feel that it is not enough to know the material well. In order to answer smart student questions, I want to be prepared in the subject well beyond what I expect to need. This might require also redoing some exercises myself. There are so many aspects to every subject: historical, cultural, there are relations to other fields. Teaching a subject requires to know as many other approaches as possible. This not only allows to chose the best way, it also allows the teacher to understand original new approaches found by students.

34.19. A teacher who teaches a course in computer science should have developed independent code. Only then, one can know how much time it can take to write code and what agony it can be if a computer program does not work yet. A piano teacher needs to be able to play the pieces which the pupil plays. The professors who taught me programming had been involved personally in projects building computers or developed computer algebra systems. All my personal piano teachers were able to play well the music pieces which I had to practice. Most of the time I felt that a good teacher should also be good at doing things and can demonstrate that regularly. This not only builds respect, it motivated me as a student.

34.20. Also important is that a student who wants to really learn a subject needs to learn it beyond the level which is needed. For example, I feel that a student who wants to master multi-variable calculus seriously should get at least to the integral theorems. While the later theorems are not used by most students later on, their understanding involves essentially every topic which appears in that course: curves, surfaces, fields, derivatives, integrals, area, volume, cross and dot products.

34.21. A mountain guide leading a group of tourists up the Swiss Matterhorn needs to master higher climbing levels. Such skills might be needed if an accident happens or the conditions become unexpectedly more difficult, for example due to weather. A tourist who wants to climb comfortably up that mountain also needs to have done some climbing before so that the climbing does not involve too much risk. There are climbing walls or boulder gardens, where one can practice various levels of climbing difficulty without danger. Being able to climb levels above what is actually needed when climbing the real mountain makes things more rewarding. The same could apply teaching. The beyond-understanding is sometimes only an ideal. I have a few times taught material that I just learned while preparing the class. I also climbed difficulty levels in the mountains which were close to the limits of my abilities and where under some less optimal conditions, the climb would have become dangerous.

34.22. It can be refreshing to **start with a blank slate**. It allows to see things from a new perspective.

34.23. When writing a new lecture or learning a new topic, it can be beneficial to avoid the library and internet from time to time and start thinking about the subject on a clean slate. When developing a lecture, it allows to ask again: what is important, why do we do what we do? Also as a student, it can help to just sit down and write down what one knows about a subject closing off any additional help for some time. Places, where one is stuck are places, where the topic has not yet been absorbed become then evident and one can with more focus revisit library or discuss it with others.

34.24. I found that starting from a blank slate can also be refreshing in research. Taking a blank paper and start developing something new can lead to surprises as it encourages to look at something old with new eyes. Mathematics education is so exciting because of the variety of teaching ideas which have been developed. This does not mean ignoring what has been done before or what others are doing.

34.25. Standing on the shoulders of giants is nice and can bring quick progress. However, when aiming for reaching some new ground, it can be helpful to climb down once in a while and try to walk without the help. Cross fertilization and building on previous work is of course needed. I myself most of the time start with some ideas which were given before or outlined by a book or syllabus. But then, after having it ready, it can help from time to time to go back to a blank slate and see whether some new possibility works better. The same happens with teaching, where ignoring previously written notes can lead to more ideas how to develop a lesson. It can also happen that one get reassured that the previous taken path was a good one.

34.26. The art of reaching the audience is finding the right **level of difficulty**, language and pre-knowledge.

34.27. Mathematics comes in different layers. Thousands of years ago, humans started to compute using sticks or pebbles. Today, the mathematical world is so huge that it is no more possible for a single person to overview all. While the nuts and bolts of mathematics like calculus or linear algebra are pretty well covered, there are also many nice popular books in mathematics which do a good job explaining more advanced topics. Examples of popular, beautiful or elegant mathematics are abundant [**1**, **3**, **13**, **14**, **17**, **21**, **23**, **23**, **24**, **32**, **36**, **52**, **61**, **66**, **78**, **84**, **85**]. We live in a great time of mathematical literature.

34.28. I also feel lucky to see that more and more also harder topics of mathematics are covered in popular culture. This can be movies which contain mathematics [**67**]. There are Mathematics exhibits like the "National museum of Mathematics in New York" or the permanent "Mathematica exhibit" at the Museum of Science in Boston. In my experience, pop culture can help in teaching. I personally collect movies also in order to be able to use it for illustrations. Sometimes, I have not found yet the right level of difficulty or abstraction in a class. A link o some pop culture can serve as a lubricant. I have been told by quite many teachers that they had been using my collection in their own class room.

34.29. We see also more and more appreciation for history and historical connections. Bringing in some historical remarks into a lecture can be helpful especially when covering more difficult topics. For Stokes theorem in multi-variable calculus, finding the right level of difficulty can be hard. Mentioning a bit of history about the theorem like [**33**, **40**, **79**] not only helps to smooth out the toughness of the topic but also can lead to a long term memory connections. It is nice to have a theorem linked to a story. In the case of Stokes theorem, one can mention for example that the result had been assigned as an exam problem by Stokes and that Maxwell was one of the students taking that exam. Maxwell was later able to write down marvelous equations which describe electromagnetic waves. When looking back myself and trying to remember some lectures from my college experience, the parts in which historical connections were woven in, are still the most memorable for me.

34.30. Sensitivity needs to be acquired and this can even for experienced teachers need constant work. It matters especially in interactive class rooms.

34.31. Teaching requires sensitivity and feel. Maybe less so in more advanced topics, where students are already prepared to absorb less refined material for example while reading research papers which by nature often are not yet polished or listening to talks which often include references to rather new results using background knowledge which is found elsewhere or which is not even published yet.

34.32. I believe that sensitivity can be trained and acquired, for example by taking feedback serious. There are the obvious sensitivity traps. Some include vocabulary. The word "trivial" for example can be devastating in mathematics. We also live in a more and more politically divided country where partisanship is more important than issues. This spills over into the classroom. We usually have students from all of the political spectrum in the same classroom.

34.33. Many different **teacher-student interactions** can work. It can depend on material, teachers and the students.

34.34. Today also some teaching methods are promoted from high school to colleges. An example is the concept of flipped classrooms, where teachers do no more teach but guide, where lectures are moved online and class time is used for discussion and work. It can make sense for some courses, but to push it universally is not a good idea in my opinion. [Added in proof of October 2020: we have now more experience in these techniques teaching happened online since March 2020. It will be important the next years to evaluate the effectiveness. My own perspectives from 2019 will certainly need revision. Our understanding of teaching might change during these months.]

34.35. As a student, I found seminars could become very intense and sometimes stressful. I found larger audience lectures more relaxing and still inspiring. Seminars were an important part of my college experience, but I liked very much also the calmer and brilliant delivery of lectures which did not feel like a competition and which allowed me to see how some great minds think and react, live and uncut. That I'm not alone with this can be backed up that there are many books which exist about meetings [**15**, **75**, **80**]. [Added in proof in October 2020: also here, the push to pure online teaching has changed things with respect to content delivery. There are some nice interactive tools like polls or collaboration jamboards which can be used quite elegantly in remote classrooms. How effective they turn out and well they can bring the students to the required level will need evaluation.]

34.36. There is a central and disputed issue of teaching and it comes up in many pedagogical discussions: how interactive should a classroom be? The answer is not easy. There are many opinions and the topic also came up at this conference. I myself found interactive classrooms in general harder to teach. My experience as a student (50 years) and as a teacher (30 years) has always been that managing an interactive classroom is difficult for teachers. If done well, it can be fantastic, but the margin of error is small. A teacher who can not lead, does not know the students, can not be proactive, can not read body language or the mood of the participants, who uses the wrong language or can not bridge different preparation levels which are present in the class, is likely to have a harder time and might fail to achieve the goal of a lecture. When advising graduate students I always encouraged to push for a high interaction

level but only so far as it is comfortable. That comfort level can very much depend on the personality of the person.

34.37. It is possible for example that group work ends up in train wreck because of student dynamics. A student might dominate and do all the work for example. There are many more things which can go wrong and lack of sensitivity (both on the student and teacher side) often play a role. The off-shot is that most people think they are good in guiding discussions in interactive classrooms, while in reality, very few really can deliver. I have also seen teachers who can do that well (also during the conference). It needs strong psychological talent and experience to read all students and distribute the workload adequately, so that not only a few students dominate while the others just follow. [Added in proof in October 2020: in online settings, the group work actually can be organized pretty well using online platorms like zoom or blackboard. In smaller groups more difficult situations appear if some students decide not to turn video like if a technical problem prevents this. So far, it has worked smoothly.]

34.38. Maybe one reason why I personally found highly interactive class rooms difficult is that I'm a rather slow thinker myself. I need time to digest something and place it into the correct spot in the landscape of knowledge. Often, I'm confused at first. Of course, this happens more frequently in more advanced topics and I constantly try to learn new things. In most new topics which I encounter, I'm completely lost at first, sometimes have not the faintest idea at first what the topic is about. This is the same situation, a student who is learning algebra, probability or calculus is in at first. Now, there are always students (or colleagues) who have spent more time with a subject and can tell the answer immediately. This can be frustrating for students in a discussion session. I have noticed such issues in inquiry based learning (IBL) conferences, I have experimented with IBL techniques for decades. It still appears to me that teaching an effective IBL lecture is hard if one wants to reach the goal. As smaller the classes, the better this goal can be achieved. It still needs (and this is from experience when seeing teachers doing it) a considerable talent, leadership skill and charisma needed to make it work.

34.39. I have already been as a student been in flipped classroom (flipped classroom means for me "no teaching by the teacher at all") as a student, where an entire hour was wasted because a hyperactive other student wanted to perform and present ineffectively without a course assistant intervening. I have seen interactive practice lectures by other teachers, where I had problems to understand at first what the subject was about. An extreme case was a practice lecture, where a teacher handed out worksheets, asked us to work together on the chalk-boards and then report. The teacher maybe said two sentences in the entire lecture and otherwise just had students report. I believe that one of the main challenges of flipped classroom settings (meaning in the strict sense of no lecture at all in the classroom) are that teachers do not feel having to prepare anymore. The effect is that teaching culture can get destroyed. Little enthusiasm is transferred if the lesson serves cold coffee, possibly brewed years ago by somebody else. It can be as cheap as teaching verbatim from a book. In my own teaching in regular classrooms, I have always been successful with Socratic teaching methods. This means regular and frequent discussions in plenum, group discussions and work guided by teacher and course assistant, but also including some lecture. [Added

in proof October 2020: also in online settings, having a good mix and variety seems to work well to foster some sort of community. Today, a variety of tools and formats are already available.]

34.40. Humor needs time, reflection and work. It can have huge advantages but its dependence on context bears risks.

34.41. Humor is a tool to ease tension and math anxiety. It can serve as an ice breaker. Humor needs training because humor is highly context sensitive. An established comedian has more leverage than a newcomer. The person itself can matter: I myself (as a Swiss) can make allow to make jokes about Switzerland or jokes about my relatively short body size (because I'm of shorter statue). It should be clear to everybody that jokes about religion, politics, body features, gender orientation or origin in general can be problematic. Even professional stand-up comedians or caricaturists have overstepped in this respect. An over-reaction can be to avoid or to forbid humor, especially in an educational or political setting. Humor however can be a lubricant which helps to ease tension and anxiety. It has been a life saver in diplomacy [**62**]. Mathematics especially has the reputation of being inaccessible and humorless. It can be difficult to use it well however. Forced humor for example almost never works.

34.42. A personal experience: in the spring of 2019 I had been asked by a Harvard comedy club to perform stand-up comedy in a faculty charity event. After they asked repetitively, I finally accepted, knowing that it will be a lot of work. I actually underestimated it even then. It needs a lot of time to come up with jokes and stories, it needs to be rehearsed and then performed well. I worked hard but it could have been much better. The experience made me also investigate a bit what the literature says: [**60**] investigates the social and cognitive benefits of humor and tries also to classify humor using a taxonomy: an example of a canned joke is a narrative, an example of irony is a self-deprecation, an example of a mockery is a parody. [**59**] stresses the links between positive emotions and education and also mentions humor as a survival tool for stressed leaders. I think it can be a survival tool also for stressed teachers and students.

35. Junior faculty

35.1. We live in a time, where the employers less and less commit to permanent workers. The word is "gig economy" [**68**]. Similarly as companies hire more and more contractors or temporary workers, universities hire more and more time-limited faculty and outsource operations like dining or infrastructure to third parties. The percentage of contingent faculty has risen from 21.7 percent in 1969 to 66.6 percent in 2009 and is now believed to be around 75 percent [**81**]. In [**11**], it is put more bluntly: "Thirty five years ago, nearly 75 percent of all college teachers were tenurable, one a quarter worked on an adjunct, part-time or non-tenurable basis. Today, those proportions are reversed".

35.2. It is a general trend that human resources are treated as less and less permanent. The reason is quite simple: it is just cheaper. Jobs change fast, skill requirements change fast. The modern worker has to constantly reinvent and adapt, possibly work two or three jobs at the same time. Both my grand fathers stayed in one profession, both my parents already reinvented themselves in their forties and branched out in

other things. In our generation, reinvention comes earlier, like in the twenties or thirties with a second reinvention in sight with 50. This might accelerate even faster in the future. The situation is complicated [**12**].

35.3. In mathematics as well as in other sciences, more and more PhD's are produced which do not have the opportunity to get into academics. The prospects of getting a tenure is not always great. The document [**30**] only publishes the immediate hiring statistics after graduation and not long term tenure data. Many brilliant graduates lose in the lottery of getting a fixed and permanent position. I consider myself extremely lucky to have been able to work in the math profession. I would have much less chance today, where hundreds of applicants line up for a position like mine and larger committees make hiring decisions. Long gone are the times, when department chairs would pilgrim to AMS conferences to hunt for good PhDs and consider themselves lucky to get one. (Ulam [**82**] writes: "*The situation was completely reversed in the late 1950ies and early 1960ies, when a lone young man fresh from school, with a brand-new Ph.D, would be surrounded by chairmen looking for young professors.*" At that time, women were excluded from the profession. We should therefore not only with nostalgia look back upon this quote of Ulam.

35.4. At Harvard, graduate students and even undergraduate course assistants are now unionized. Some of the staff already belongs to unions. There are efforts to have tenured professors unionized but the group of non-permanent, so-called non-ladder faculty have not much protection yet, certainly no job security and there are also no plans for a union. We had in the fall of 2019 the situation that graduate students and some course assistants were on strike. Of course, the additional work-load had been pushed to the remaining teaching faculty. We internally supported the strike, but the story illustrates that non-ladder faculty are probably the least organized of a university today [**81**]. The unionization topic has come up also in the conference. It is a complex issue [**5**].

35.5. The terminology which is used for non-permanent teaching faculty ranges wide, it goes from "junk faculty", "contingent faculty" to "non-ladder faculty". At Harvard, part of this group are "preceptors", part of "teaching faculty". I like also the name "junior faculty" as it is a profession which keeps you young! We are sometimes also considered part of the "invisible university" as it is usually a postdoc position. We are present in all higher education and play an important role in teaching and administration. [Note added in proof: a referee has pointed out that the terminology VITAL faculty seems increasingly be used. It stands for Visitors, Instructors, TAs, Adjuncts, and Lecturers. This is actually a nice name and it feels very fitting because this group is a truely vital part for higher education.]

35.6. While the profession has still low prestige, it comes with upsides. There is no stress nor pressure to do research. There is a wide variety of work with like administrative, teaching, technology or management. There is flexible work time and a stimulating academic environment with many colleagues sharing similar passions and interests. For the most part, there is also autonomy.

35.7. The downsides are minor, but they need to be noted and one has to be aware of them when choosing the profession: first of all one is replaceable. But this is not unusual, even top leaders in a corporation can be replaced. The obvious difference is not having a "golden parachute". There is also little political power as there are usually no voting rights. The advantage is that less meetings that have to be attended. Teacher positions often come with low prestige. This is shared with assistant or postdoc positions. The low status has also advantages as it selects folks who do not care so much about titles and huge salaries but are excited about the subject and love what they are doing.

35.8. Also junior faculty suffer from the "fall of faculty" [**28**, **47**]. Maybe it has been accelerated by electronic performance programs, but administration in most recent years gained more and more influence in universities. As faculty want to do less administration, they also lose control. The provocative book [**11**] starts with some thoughts of Albert Einstein, who in 1949 urged intellectual workers to secure their influence in the political field because: *"The intellectual worker, due to his lack of organization, is less well protected against arbitrariness and exploitation than a member of any other calling".* Since the time of Einstein much has changed. It does not only apply to the intellectual worker, but in general for all of work. It is a challenge and not easy to navigate this. Historians try to use the past to look ahead [**34**].

36. Personal Words of wisdom

36.1. As mentioned in the introduction, the following personal words are not meant to feel condescending but hope to be helpful. There is lots of professional advice coming from experts [**22**, **29**, **50**], but it could also be useful when it comes from ordinary folks like me.

36.2. Happiness is achievement minus expectation.

This is a variant of "Happiness = Reality - Expectations" [**34**]. Great expectations, maybe fueled by early success, usually are crushed and lead to misery. Rich people need more riches and worry about losing it, beautiful people fret getting old, tenured faculty worry about getting good graduate students and if they get them, being able to provide them with jobs. Fame and fortune can also lead to trouble. Countless celebrity stories illustrate this. It does not mean that one should not try achieve the best possible, but it can also be helpful to temper expectations. There is a lot which is not under our control.

36.3. Be prepared to hack your value system.

What counts is not what other people think but how you feel when doing the work on a daily basis. If there are 2-3 hours a week, where one feels under appreciated and 100 hours a week are filled with fulfillment and achievements, then one is a lucky person. What is important is to be able to hack the value system. Kottke [**46**] once formulated a **theorem of Lebowski on machine super intelligence** "No AI will bother after hacking its own reward function.". Of course this was meant with a tongue in the cheek but there is some truth in it and some super intelligent mathematicians have shown that it is possible.

36.4. Mind the confirmation bias.

Confirmation bias is well documented. There is negative bias as well as positive bias. An example of positive bias is the **Dr Fox experiment**, which has been done as part of a PhD by J.E. Ware in 1974 [**83**]. There are consequences of confirmation bias: an unknown or unaffiliated mathematician has a harder time to publish stuff. This can also be a blessing as one is evaluated for the work and not whether it is a friendly colleague who referees the paper. It can be brutal as it is possible not even to get a referee report, but the advantages are there: having not to revise a paper frees up time to explore other paths. And also, there is little danger of being plagiarized or followed and raced to a new result. Doing some work away from the spot light is actually very relaxing.

36.5. Disappointments are opportunities.

Having not reached the goals one has been dreaming of, also comes with advantages. I definitely do not suffer from the impostor syndrome [**87**]. I do not have to worry about hunting for recognition or being hunted for committee or editorial work. It actually is amazing how fast any disappointment fades away. I myself have passed lots of disappointments but always, there has been also something good about it. One of the advantages is "remaining hungry". The stakes are lower if success is not expected. Also, if one is at the bottom, it can only go up. A rich person always lives with the worries to be robbed. If one has less possessions, there is less to worry about.

36.6. Doing can be more fulfilling than managing.

There is the "shop manager paradox": the daily job of a higher ranking manager can often be much more stressful and boring than the job of a person who actually creates and does things. The reason is that seeing things grow first hand can give satisfaction. Creating a dish or programming a piece of software can be more rewarding than supervise somebody to do it. This is of course a personal thing. Some are happier when doing stuff, others are better with telling others what to do and see things grow from a larger perspective. Some like to manage the achievements of the entire group, others enjoy doing it and contribute to the group.

36.7. Appreciate the small.

A lot of satisfaction can come from teaching, even if it is a lecture has been done already many times before. Also the preparation can be fun. One might have to write a little program for illustration purposes or then have to work out administrative issues. It is the satisfaction which a mason gets when building a wall, or a designer has when creating a program or an actor when playing a scene. Some creative minds move on and become owners of a shop or become a movie director or even a movie producer. This comes with more prestige but it also can come with more frustration as the success can depend on factors on which are no more under direct influence any more. A software company owner needs good programmers, a director needs a good film crew and technicians etc. In general, the probability to do something meaningful and personally

satisfying is bigger when scoring in small things but it is also less rewarded financially for example. This makes sense as it comes also with less risk.

36.8. Experiment and learn new things.

To be at the bottom of the ladder makes you feel that there is opportunity to grow, to branch out, try other things. This is harder when not only your own venture is at stake but a larger structure must work. In a tenured situation with graduate students, it is hard to change the subject. There are obligations to a field of colleagues, to graduate students, expectations. Only few (usually only the best) can afford to completely change field. There are examples of fields medalists who have done that. Remarkably, it is also relatively easy for non-ladder faculty who are not hired as research mathematician but as teaching faculty. I myself enjoyed learning to work in new areas of mathematics and be in situations where the learning curve is still steep as things are new.

36.9. Be aware of the age myth.

There is a myth that mathematics is a "young man's game". It is twice wrong as mathematics is neither a game for men, nor a game for the young. The source of this misconception can be traced back to G.H. Hardy [**35**], who wrote in 1940 a rather depressive essay about a year after he had a heart attack (1939). Jean Dieudonné, considers a person to be a mathematician, if he or she has proven a nontrivial theorem, and points out that Kronecker, Cartan, Siegel, Weil, Leray or Gelfand have continued to prove fine theorems after the age of sixty. Many important textbooks (also containing original material) have been written later in life: In Ivor Grattan-Guinness book "The Rainbow of Mathematics" [**31**] features the ages of the authors. Galileo did great things with 74, Napier with 64, Bürgi with 68, Briggs with 63. An early study [**77**] already point out why there is a bias: many bright young mathematicians do not continue, possibly because they did not get a permanent job. In fact, many mathematicians do their best work in mature years if they have the chance. The amount of mathematics which has been produced still grows exponentially. Most theorems proven as a young mathematician will turn out to be already known. It needs time to get experience and see what is known and what is not. A more experienced mathematician can navigate the immense knowledge sea better. Finally, mathematics is probably the only science, where senior mentors help to lift their fledglings, often without appearing as coauthors. In other sciences, the first name of the paper might go to the principal investigator, even so the later might not have contributed much besides managing the project. In math, first works are often done on the shoulders of giants or mentors. This distorts the picture and suggests that creative work can only be done by the young.

36.10. What do you care what others think of you?

There are always others who are better. One can rub this in by giving away prizes, grades or student evaluations. Of course, evaluation is important, but once an evaluation system is in place, it is often taken as the gold standard. Especially damaging are the side effects of awards. It is something one is hardly aware of. While awards encourage a few, it can discourage the majority. There is much collateral damage coming

from prizes. The effect is that in the immediate neighborhood of a prize winner, ambition is blown out. The oxygen has been sucked up by the nearby fire. This happens especially in early years, like in primary and middle school, maybe early high school. Later in high school or in college the interests have diversified already, so that it does not matter any more so much. Higher levels of depression are present in student populations [**19**, **53**]. The increase from 2007 to 2018 among US undergraduates is dramatic [**55**]. And it is not only students who suffer from the "impostor syndrome" and ask themselves: "do I really belong in this school?" It can help already to be aware of such mechanisms [**87**].

36.11. We all have the same hardware.

Why is that we observe large cognitive differences even so all humans have give or less the same brain structure? This has been investigated for a long time already like by Piaget [**65**] or more contemporary psychologists [**26**]. There is also the more computer scientist point of view [**56**]. A computer scientist would say that we all have the same CPU, RAM and clock-speed. How come then that some mathematicians are perceived to have multiple times better abilities than others even so they operate with the same hardware? Practical computer science tells us that not only the hardware but also the operating system matters for effectively running programs. The operating system is also software which can be tweaked. It suggests that not only mathematical ability but **mathematical talent** (what ever this means) can be improved through education. What we have not yet figured out is how to tweak the operating system more effectively, teach how to learn, and to teach how to search for new mathematics. It also leads to the question what does mathematical ability means. Certainly, there is not only a computer science but also social, historical, economic and cultural context.

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Turning Coffee into Unions: Mathematicians and Collective Bargaining

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In memory of Lauren Berlant

> *We will get nowhere without a wholesome group consciousness. Our worst troubles as a profession arise from unwarranted assumptions of superiority on the one hand coupled with a too ready acquiescence on the other.* —Guido Marx, in a 1914 letter to John Dewey (see [**10**,**21**])

I am easily embarrassed and rather uncoordinated, so dancing in public is very much not my thing. Yet, just over a year ago as I write this, I found myself doing the Hokey Pokey while walking in a circle on the University of Chicago's main quad. The context is that I was part of a group keeping its spirits up after several hours in the rain, during a three-day industrial action by Graduate Students United (GSU), the university's democratically elected but so-far unrecognized graduate student worker union. I was there as one of many supportive faculty members, and also in my role as president of the University of Chicago Advocacy Chapter of the American Association of University Professors (AAUP).

I mention this incident in part to lay my cards on the table from the get-go. This is not an academic paper, but an activist's meditation (and exhortation) on academic unionization and modes of resistance to the range of phenomena that are often referred

I thank Emily Riehl for inviting me to the workshop at which I gave the talk on which this essay is based, and thank her and the other participants for making it a delightful and deeply thought-provoking experience. I also thank Larisa Reznik for reading a draft of this essay and making many extended comments, which greatly helped me shape several sections, and to Yali Amit, Damir Dzhafarov, and Emily Riehl for their careful reading and comments on a later version. I have had the privilege to be part of a Critical University Studies reading group with several brilliant colleagues who have almost infinitely more knowledge and experience than me in this and related areas, and from whom I have learned a great deal. I am grateful for their willingness to accept a mathematician among their number. Several of the texts I cite in this essay are ones I learned about from them. I am equally grateful for the many illuminating conversations I have had with fellow workers associated with the University of Chicago Advocacy Chapter of the AAUP, the UChicago Labor Council, and other organizations connected with the Labor Council, such as Graduate Students United and UChicago Faculty Forward.

For nonmathematicians: the title comes from Alfréd Rényi's "A mathematician is a device for turning coffee into theorems."

to as "academic corporatization", particularly focused on the perspective of mathematicians and mathematics departments in the United States.[1] It is largely based on my talk at the workshop "A Conversation on Professional Norms in Mathematics", organized by Emily Riehl at Johns Hopkins University, Baltimore, MD, Sept. 20–22, 2019. At the same time, it is being written during the 2020 COVID-19 pandemic, as college and university workers are facing layoffs, cuts to benefits, health risks created by working conditions in which they have had little or no say, and a number of other issues that, while brought about in part by extraordinary circumstances, should remind us of the precarity and dependence so many of these workers face even at the best of times. It is also being written during the 2020 Black Lives Matter protests, which have highlighted just how much work there is to do in achieving even a modicum of equity and justice in the United States and throughout the world, and how much the structural flaws of large institutions can stand in the way of this work. Colleges and universities are very much among such institutions, their stated commitments notwithstanding, and their flaws cash out in many ways, one of which is an entrenched opposition to power-sharing in general, and unionization in particular. I want to make a case to mathematicians in the academy that we should not succumb to this antidemocratic mode of thinking ourselves, and indeed should actively fight it, for our sake, that of our community, and that of those who make a community so highly dependent on complex social structures possible.

I believe in a fully unionized academy: faculty, student workers, other academic staff, and of course nonacademic staff who are as much a part of the endeavor as we are.[2] I believe in solidarity among these groups and across institutions. I believe in all of this happening as part of a broad labor movement whose revitalization in this country is long overdue. And I believe in this 21st century labor movement as part of the even broader struggle for social, racial, economic, and environmental justice. This is a struggle with many goals, which we should never allow to be pitted against each other or put into competition. They should form a "sphere, whose center is everywhere and whose circumference nowhere" (see [**7**]). But the work itself does need solid centers. It needs robust instruments of resistance, collective action, power-sharing, and democratization. My excitement about the potential that labor unions have to play this kind of role within a context of cooperation and solidarity, to the benefit of all of us, is what animates this essay.

Before going into a discussion of what there is to be gained, and what losses there are to be avoided, by collective bargaining in the academy, let me mention some of the barriers and opposition it has faced. Both legal blocks and internal resistance by academics themselves stem at least in part from misperceptions about the organization and impact of academic institutions. Much has been written and said about these issues, but as a first step toward understanding just how distorted is the picture of such institutions as largely faculty-led, and committed to the careful stewardship of limited resources for the greater common good, I highly recommend watching the recent panel

[1] Or really, to be fully honest, on that of those at institutions somewhat like mine, only because that is what I know best. I will use the University of Chicago as a running example, but hope there are things here that speak to a broader context, and to academic workers other than mathematicians.

[2] It is an unfortunate flaw in even some of the better writing on collective action in the academy to leave out the importance of solidarity between academic and nonacademic workers at colleges and universities.

[**30**] on "Austerity, Racial Capitalism, and Universities",[3] which also discusses the role of both academic and nonacademic unions in exceptionally thoughtful ways.

Academic unionization at private institutions in the United States has a complicated history in labor law.[4] Consider graduate student worker unionization. The makeup of the National Labor Relations Board (NLRB)—an independent federal agency tasked with safeguarding employees' rights to organize and determine whether to form unions, and to address unfair labor practice issues in the private sector—changes with appointments made by different presidents, and thus so do its interpretations of the law. In 2004, in a case involving Brown University [**37**], the NLRB sided with the argument universities usually make that graduate student workers should not be considered employees under the terms of the National Labor Relations Act (NLRA). In 2016, the NLRB reversed this decision in a case involving Columbia University [**38**], saying that the Brown case "deprived an entire category of workers of the protections of the Act without a convincing justification." This development led to several union elections, followed by challenges by universities. When it became clear that Trump administration appointees to the NLRB were going to use these cases to overthrow the Columbia decision and set precedent, these unions dropped their cases, and have since then sought voluntary recognition, in some cases successfully, for example at Brown, Georgetown, and Harvard, where union members recently ratified contracts [**28**, **29**, **37**]. (More recently, the NLRB declared its intention to reverse the Columbia decision via rulemaking rather than in connection with a particular case.)

There is no doubt that at universities that are at least minimally functional, the work that graduate students do as instructors, teaching assistants, and research assistants is frequently beneficial to their training. (I often learn a great deal from my teaching too. I think many of us are familiar with the idea of learning a subject by teaching it, for instance.) However, that fact in no way means that what they do is not at the same time labor that is absolutely essential to the functioning of these universities. My own department's teaching program could not function in any form even remotely resembling its current one without relying on graduate student labor. Indeed, a student who comes in completely decided to go into industry and never teach after obtaining their PhD will have to teach just as much as any other student in the department. This is in no way to minimize the work that my colleagues who run our undergraduate program do to help prepare and support our graduate student instructors. Many employers offer extensive training and support to workers in demanding positions. Many employees take advantage of this training to move to better positions with other employers. These facts in no way make these employees nonworkers. I have had several graduate students as College Fellows, a position that entails being a teaching assistant in a training context, which among other things asks instructors to act as teaching mentors to their fellows. I hope that these fellows have gotten something from the experience that was of help when they started teaching their own classes. However, what I know for absolutely certain is just how much they have helped me in running my courses. I find absolutely ludicrous any suggestion that, because they were students, they were not at the same time workers, whose labor contributed to the university's operation in the

[3]This panel, held by Scholars for Social Justice, was moderated by Sarah Haley, and featured Davarian Baldwin, Frank Deale, Destin Jenkins, and Barbara Vereen.

[4]The legal issues at public institutions are significantly different. Particular legal issues also pertain to religiously-affiliated institutions.

same way that mine does. And this is not even to speak of the students who teach their own courses.

Graduate student workers' limited-term employment is also no argument against their unionization. Indeed, quite the contrary. In "STEM is Overrated" [**25**], when discussing Louis Hyman's *Temp: How American Work, American Business, and the American Dream Became Temporary* [**12**], Caitlin Zaloom writes that,

> [b]eginning in the 1970s, corporate heads and their consultants began to look for short-term profits, cutting their commitments to their employees. Workers who might stay for years or decades required promotions and benefits and were protected by unions. Disposing of expensive workers became a key to meeting profit targets. In their place, corporations began to rely on short-term employees who would stay for the job at hand and then leave.

The unionization of temporary workers is thus a key part of the contemporary labor movement, and without it, the NLRA will be weakened to something well short of its intent "to protect the rights of employees and employers, to encourage collective bargaining, and to curtail certain private sector labor and management practices, which can harm the general welfare of workers, businesses and the U.S. economy." [**31**]

Tenured and tenure-track faculty are in an even more difficult position, given the 1980 Yeshiva University case [**39**] in which the Supreme Court held that "[t]he University's full-time faculty members are managerial employees excluded from the [National Labor Relations] Act's coverage." As explained more fully in the syllabus (a summary at the head of the decision), the majority held that

> [t]he controlling consideration is that the faculty exercises authority which in any other context unquestionably would be managerial, its authority in academic matters being absolute. The faculty's professional interests—as applied to governance at a university like Yeshiva which depends on the professional judgment of its faculty to formulate and apply policies—cannot be separated from those of the institution, and thus it cannot be said that a faculty member exercising independent judgment acts primarily in his own interest and does not represent the interest of his employer.[5]

This decision is well worth reading for those interested in the perception of academics and our institutions, particularly as much that is said there remains a live issue four decades down the line. Indeed, the decision is too germane to this discussion to forgo quoting at some length from the ruling, delivered by Justice Lewis F. Powell, Jr. and joined by four other justices; and the dissent, filed by Justice William J. Brennan, Jr. and joined by three other justices.[6] They set the stage for this discussion rather well. In addition to the legal question itself, the decision rehearses debates about the relationships between academics and their institutions, and about the very nature of these institutions as they exist in the real world, rather than the fantasy one of college brochures and official institutional histories. The dissent in particular points to

[5]Here and below, I have retained the gendered language of the original legal document, which unfortunately infests even some of the otherwise admirable passages from the dissent. I wish I could say that this is a practice the Supreme Court has long since abandoned.

[6]In all cases, I will omit footnotes from quotations.

the benefits of collective bargaining in the academy not only for individual academic workers, but for the entire endeavor, especially in the context of education and research as a business, in a way that makes it clear that academic unionization is well within the spirit of the labor protections envisioned by the NLRA, and should be well within the spirit of the political commitments of those who support organized labor as a whole.[7]

The contention of the ruling is not that faculty supervise other employees (indeed, the Court chose not to resolve the issue of the supervisory status of faculty), but that they are managerial employees, defined as those who "formulate and effectuate management policies by expressing and making operative the decisions of their employer." (Here the ruling is quoting an earlier Supreme Court decision, which itself is quoting an NLRB ruling from the 1940s.) Part of this classification has to do with the ruling's concurrence with the Court of Appeals for the Second Circuit, whose own ruling in this case it upheld, in the Court of Appeals' finding that the NLRB

> [...] had ignored "the extensive control of Yeshiva's faculty" over academic and personnel decisions as well as "the crucial role of the full-time faculty in determining other central policies of the institutions." [...] the faculty are, "in effect, substantially and pervasively operating the enterprise." [Quotations here are from the decision of the Court of Appeals.]

And later:

> To the extent the industrial analogy applies, the faculty determines within each school the product to be produced, the terms upon which it will be offered, and the customers who will be served.

Now, I don't know what conditions actually obtained at Yeshiva University forty years ago, but I expect this description of faculty control will seem rather alien to many at institutions to which the Yeshiva decision still applies (even if the product/customer analogy unfortunately might feel much closer to home). As the dissent puts it,

> [...] the Court's vision is clouded by its failure fully to discern and comprehend the nature of the faculty's role in university governance.
>
> Unlike the purely hierarchical decisionmaking structure that prevails in the typical industrial organization, the bureaucratic foundation of most "mature" universities is characterized by dual authority systems. The primary decisional network is hierarchical in nature: Authority is lodged in the administration, and a formal chain of command runs from a lay governing board down through university officers to individual faculty members and students. At the same time, there exists a parallel professional network, in which formal mechanisms have been created to bring the expertise of the faculty into the decisionmaking process. [...]
>
> [...]
>
> And while the administration may attempt to defer to the faculty's competence whenever possible, it must and does apply its own distinct perspective to those recommendations, a perspective that is

[7]It is not irrelevant that Powell is usually regarded as having been a business-friendly conservative, while Brennan is regarded as having been one of the Court's leading liberal voices.

> based on fiscal and other managerial policies which the faculty has no part in developing. The University always retains the ultimate decisionmaking authority, [...] and the administration gives what weight and import to the faculty's collective judgment as it chooses and deems consistent with its own perception of the institution's needs and objectives.

With the passing of the years, even the more realistic picture this passage paints seems like a significant overstatement of the role of faculty at many institutions. At how many institutions nowadays, especially larger ones, does the administration "attempt to defer to the faculty's competence whenever possible", unless that is taken to mean, "whenever none of the many other things it thinks of as more important apply"?

However, even more jarring to me is the idea of "alignment with management". According to the ruling, the exemption from coverage under the NLRA for managerial employees comes out of the concern "[t]hat an employer is entitled to the undivided loyalty of its representatives." As the ruling goes on to say,

> [m]anagerial employees must exercise discretion within, or even independently of, established employer policy and must be aligned with management. [...] normally an employee may be excluded as managerial only if he represents management interests by taking or recommending discretionary actions that effectively control or implement employer policy.

As the dissent says,

> [...] [t]he touchstone of managerial status is thus an alliance with management, and the pivotal inquiry is whether the employee in performing his duties represents his own interests or those of his employer. If his actions are undertaken for the purpose of implementing the employer's policies, then he is accountable to management and may be subject to conflicting loyalties. But if the employee is acting only on his own behalf and in his own interest, he is covered under the Act and is entitled to the benefits of collective bargaining.

The majority clearly believed the former was the case: "In fact, the faculty's professional interests—as applied to governance at a university like Yeshiva—cannot be separated from those of the institution." The ruling even goes as far as to say that "[t]he large measure of independence enjoyed by faculty members can only increase the danger that divided loyalty will lead to those harms that the Board traditionally has sought to prevent." Which seems to me like "damned if you do and damned if you don't" at its worst.

I can't be the only one who, with the concept of academic freedom firmly in mind, has no more articulate first response than "Say what?!" The dissent puts it a bit more fluently:

> Unlike industrial supervisors and managers, university professors are not hired to "make operative" the policies and decisions of their employer. Nor are they retained on the condition that their interests will correspond to those of the university administration. Indeed, the notion that a faculty member's professional competence could depend

> on his undivided loyalty to management is antithetical to the whole concept of academic freedom. Faculty members are judged by their employer on the quality of their teaching and scholarship, not on the compatibility of their advice with administration policy.

And later on:

> [...] The very fact that Yeshiva's faculty has voted for the Union to serve as its representative in future negotiations with the administration indicates that the faculty does not perceive its interests to be aligned with those of management. Indeed, on the precise topics which are specified as mandatory subjects of collective bargaining—wages, hours, and other terms and conditions of employment—the interests of teacher and administrator are often diametrically opposed.

Near the end, the dissent gives a picture of the state of the contemporary university that is remarkably resonant with our current condition, but even more remarkably, still not universally acknowledged:

> Finally, the Court's perception of the Yeshiva faculty's status is distorted by the rose-colored lens through which it views the governance structure of the modern-day university. The Court's conclusion that the faculty's professional interests are indistinguishable from those of the administration is bottomed on an idealized model of collegial decisionmaking that is a vestige of the great medieval university. But the university of today bears little resemblance to the "community of scholars" of yesteryear. Education has become "big business," and the task of operating the university enterprise has been transferred from the faculty to an autonomous administration, which faces the same pressures to cut costs and increase efficiencies that confront any large industrial organization. The past decade of budgetary cutbacks, declining enrollments, reductions in faculty appointments, curtailment of academic programs, and increasing calls for accountability to alumni and other special interest groups has only added to the erosion of the faculty's role in the institution's decisionmaking process.
>
> These economic exigencies have also exacerbated the tensions in university labor relations, as the faculty and administration more and more frequently find themselves advocating conflicting positions not only on issues of compensation, job security, and working conditions, but even on subjects formerly thought to be the faculty's prerogative. [...]

Despite the ever-increasing aptness of the dissent's view of the academy, the Yeshiva ruling still holds. The Obama-era NLRB tried to limit its applicability in 2014, in a case involving Pacific Lutheran University [**40**], with a decision that explicitly mentions corporatization and the further erosion of the role of faculty in decision-making, but more recently the Court of Appeals for the D.C. Circuit made the application of the Pacific Lutheran decision problematic, and opened the possibility for the NLRB to reverse it entirely, in a case involving the University of Southern California [**41**].

The legal aspect is only part of the equation, however. Overcoming internal resistance to academic unionization is also a significant challenge. As Rebecca Kolins Givan puts it in "Acquiescent No More" [**10**], when discussing the passage used as an epigraph to this essay (which I will touch on again further below), "the persistent problem of a professional culture of faculty acquiescence, as Guido Marx's words demonstrate, long predated the legal denial of the faculty's collective-bargaining rights." Even if we might not see our own institutional arrangements accurately reflected in the Yeshiva decision's funhouse mirror version of academia, many of us still can't quite think of ourselves as workers, at least not the kind who unionize. As Paula M. Krebs puts it when discussing "The Faculty-Staff Divide" [**16**], we inhabit "a culture that encourages us to see ourselves as thinkers rather than as workers." (A particularly terrible application of the excluded middle…)

The most common version of this attitude among mathematicians, or perhaps among academics in general, might well be an allergy to engaging with such issues at all. "I just do math and don't get involved in academic politics." Collectively, one result of this attitude has been the sacrifice of our democratic voice to the convenience of avoiding responsibility, even at the cost of giving up decision-making power. This essay is an argument that one *should* get involved, indeed, that getting involved is essential to doing one's work responsibly, and for many academic workers, mathematicians included, might well be essential to continuing to be able to do this work at all.

But there are also more active forms of resistance to thinking of what we do as labor. We see ourselves as independent agents, who might draw salaries but are surely not "employees". We are proud that "herding cats" is often used in connection with trying to get us to do anything in concert. We see the protections of academic freedom, real or imagined, as distinguishing us from workers who can be fired for not toeing the company line. And we are at the vanguard of the modern call to love one's job, to see it as a calling that can only be diminished by engaging in the same kind of collective bargaining as a factory worker.[8] Is it seemly to worry about the nutritional content of the sacramental bread?

In "Down with Love: Feminist Critique and the New Ideologies of Work" [**23**], Kathi Weeks notes that "today management discourse seems to be obsessed with love and happiness", and draws on the work of 1970s feminist scholars to perform a remarkable critique of how the ideology of romantic love is being used "to recruit all waged workers into a more intimate relationship with work", with all the attendant benefits that this one-sided attachment brings to corporations that are under no obligation to love their workers back. As she says,

[8]Administrations like to play on academics' sense of remoteness and specialness by making much of the idea of unions as outside entities with no expertise on issues central to our precious little world. There is a rhetorical trick here in equating the union with the supporting national organization, rather than the local, whose leaders and members are fully part of this world. There is also a sly nod to academic elitism, which relies on the historical names of unions such as the United Auto Workers, and assumes academics will react with disgust, because, after all, "we make knowledge, not cars"… The lack of content in this argument was put into sharp relief for me when my own institution's administration "copy-pasted" this same argument when writing to faculty about GSU, which at the time was affiliated with the American Federation of Teachers and the AAUP. Arguing that these organizations have no experience with educational matters is at best bizarre. Of course, nothing is ever said about the academic experience of the boards of trustees and analogous entities that have far more power than any union could hope for.

> more and more of workers' subjectivities become folded into and fused with their identity as workers. To configure work as the center of our identity requires a reconfiguration of the self in its relationship to work. This is facilitated by the fact that, mimicking the unbounded qualities of household care work, in the contemporary economy the borders that were once thought to separate waged work from non-work time, spaces, practices, and relations are widely acknowledged to have broken down. Waged work and its values have thus come to dominate ever more of our time and energy.

I think many of us will see ourselves or our colleagues in this description. In a way, the rest of the world has just caught up with the academy's ethos that one's profession should be in essence a romantic attachment, and one that calls for a total commitment, even if the rest of the world has not quite yet gotten to the point of denying like some of us that what we do is even "really work" at all. This ethos also plays a part in views of graduate student worker unionization. If love is what binds us, and graduate students are "us in training", then thinking of anything they do as labor subject to collective bargaining is a category mistake, and an unseemly one at that, like suing one's parents for an allowance raise. This is also an ethos exceptionally ripe for exploitation, and even on those grounds alone we should be wary of adopting it uncritically.

I want to argue here that, more and more, this whole view is a fantasy that we can't afford ourselves,[9] and that also comes at far too high a cost to others. This is not to say that we cannot enjoy what we do, even love what we do, but that this enjoyment should not preclude us from seeing clearly that, whatever else we are, we are also academic laborers, part of a collective whose relationships to the institutions within which we perform this labor are not nearly as simple and secure as we might like them to be. And it should never cause us to look through the "rose-colored lens" mentioned in the Yeshiva dissent when viewing these institutions.

What I will not argue is that this is an easy perspective to come to for many of us, given the apprenticeship model of graduate education and the sheer wall of propaganda academic institutions put out into the ambient culture.[10] As a graduate student at Cornell in the late 1990s, my own view of academic unionization was definitely clouded by some of the misconceptions I'm talking about here. I don't really want to engage in the rhetoric of "Yeah, I used to think that, but then I grew up", but I can't deny that to some extent that is how I feel when I hear some people speak against academic unionization. I will say, though, that I'm not taking credit for anything here. It took the help of many colleagues, some whom I have gotten to know, some whom I know only through their writing, for me to change my mindset. I am grateful to them and their faith in the notion that education is a process of constant engagement, and of being willing to live with the idea that the first hundred people one talks to might not listen, but the hundred-and-first might.

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[9] To be honest, I am privileged enough to have the kind of position that would likely allow me to preserve this fantasy of being an island unto myself in a big sea of love for the next couple of decades until I reach retirement age, without much personal cost, as long as I were willing to stick my fingers in my ears and not listen to any voices outside my little domain. But even this privilege is predicated on a kind of stability that might soon become a casualty of political and environmental crises.

[10] Some would rather call it advertising. In my native language of Portuguese, the two words are the same.

Even the concept of solidarity has to be learned by those with the luxury never to have had to rely on it. This fact becomes clear when colleagues in a department that feels it takes good care of its graduate students express surprise that these students would join picket lines along with those in other, less well-resourced departments, or when administrators try to drive a wedge between full-time and part-time non-tenure-track faculty during contract negotiations. The epigraph to this essay is, as explained in [**10**, **21**], from a letter by Guido Marx, an engineering professor at Stanford and member of the organizing committee for the AAUP's founding meeting, to John Dewey, who would become its first president. Marx was arguing for the inclusion of assistant professors (who at that time were in many ways contingent faculty) in the AAUP's membership. He was unsuccessful at the time, but I am glad to say that the AAUP is now open to all academics, including graduate students, in recognition of Marx's prescient belief that "our organization gains its strength for accomplishment through its numbers as well as their representative character [...] If this movement is to get anywhere it should be firmly based on a philosophy of inclusiveness and cooperation and not exclusiveness."

I would also like to say something about the belief many of us have that we are fortunate to be paid to do something we would want to do anyway, and should feel nothing but gratitude and obligation towards our institutions. To the extent that we feel privileged to do what we do, it is essential to understand that our work depends on a network of labor to which we do a great disservice if we stand apart from it. The "thinkers rather than workers mentality" can "prevent us from understanding much of what makes our research, and especially our teaching, possible. We are encouraged to see the workers around us as there to make our jobs easier, rather than as fellow employees of a nonprofit corporation with its own corporate culture." [**16**] We also need to understand that we do this work within individual institutions and an overall academic system that have both greatly benefited from and helped create a host of inequalities. (Again, I recommend watching [**30**].) So any sense of gratitude we might have ought to be expressed by a commitment to stand in solidarity with those who make it possible directly, as well as those who have been hurt and disadvantaged by this process. When I call my institution on its failures to promote economic and racial justice, when I support academic and nonacademic unions at my institution, as well as student and community organizations fighting against some of the inequities that it has helped maintain, I am doing it *because*, not despite of how fortunate I feel to be able to make a good living proving theorems and teaching highly-engaged students.[11]

Academics' sense of mission can also get in its own way. At the time of the industrial action with which I opened this essay, a colleague asked whether those of us supporting the action took for granted that there was a protected right not to cross picket lines, and hence not teach in the case of a strike. They themselves did not feel they had such a right, but did feel a responsibility to students in their classes, and indirectly to their parents. My response was to say that, in addition to the fact that even

[11]We might have something to learn here from professional athletes, who have seen the benefits of collective bargaining despite not being what one might think of as the typical kinds of workers associated with labor unions. A few days ago as I write, WNBA and NBA players, some of whom make even more money than top university administrators, went on strike not because of their own working conditions, but as a way to protest and bring attention to the police shooting of Jacob Blake in Kenosha, Wisconsin.

highly responsible instructors will occasionally cancel classes for reasons far less important than the right of workers to have a say in their working conditions, there are many professions that come with great responsibility to others, some greater than our own, as in the case of medical professionals, but if we are to say that this should always come before the struggle for workers' rights, and should prevent us from joining labor actions or refusing to cross picket lines, then that is the end of the labor movement. I do believe I have the right, indeed the responsibility, to do my part in the struggle for the rights of workers at my university. (I am not speaking here of my legal rights, which are complicated, but of moral ones.) But even more to the point here, instructor working conditions are student learning conditions. I have supported striking nurses at the hospital at which I get my medical care, not only because I believe in the rightness of their demands, but also because I know that many of these demands are aimed at improving patient care. I feel the same about academic unions vis-à-vis students.

All of these challenges, legal and sociological, cannot be ignored. However: Laws can be changed, and organizing toward large-scale unionization can include political action to affect legislation. (As I write, for instance, the "Protecting the Right to Organize (PRO) Act", a large-scale piece of labor reform legislation, has passed the House of Representatives.) The Yeshiva decision noted that "[t]here is no evidence that Congress has considered whether a university faculty may organize for collective bargaining under the [NLRA]." There is no reason that must remain the case. More importantly, unionization efforts do not have to depend on the NLRB and the courts. Minds can also be changed. Colleges and universities can voluntarily recognize unions, and faculty can pressure them in that direction. It is fully consistent with these institutions' stated missions, and their accountability to society in general rather than to stockholders, to do so. To think that to voluntarily recognize a union is to let an enemy into the house is a failure of the imagination, an inability to recognize the mutually beneficial nature of democratic power-sharing. Collective bargaining is inherently oppositional only when institutions are inherently exploitative. Whether academic institutions are that or not at the moment, they should certainly wish not to be.

The Yeshiva dissent clearly recognizes that what we should be talking about is a collaborative arrangement, one meant to avoid rather than create conflict. The dissent sees a basic alignment of interests, and goes on to say that "[d]ifferences of opinion and emphasis may develop, however, on exactly how to devote the institution's resources to achieve those goals. When these disagreements surface, the national labor laws contemplate their resolution through the peaceful process of collective bargaining." If an institution is not too far gone down a road that we cannot countenance, this is the right attitude. And if it is that far gone, then of course that's when we *really* need the legal protection of unionization. Basically, as the dissent says, when discussing an increase in collective bargaining agreements at the time, which had led to over 130,000 academic workers being unionized, "[t]he upsurge in the incidence of collective bargaining has generally been attributed to the faculty's desire to use the process as a countervailing force against increased administrative power and to ensure that the ideals of the academic community are actually practiced." Near the end of the dissent, though, comes a warning:

> Today's decision, however, threatens to eliminate much of the administration's incentive to resolve its disputes with the faculty through

> open discussion and mutual agreement. By its overbroad and unwarranted interpretation of the managerial exclusion, the Court denies the faculty the protections of the NLRA and, in so doing, removes whatever deterrent value the Act's availability may offer against unreasonable administrative conduct. Rather than promoting the Act's objective of funneling dissension between employers and employees into collective bargaining, the Court's decision undermines that goal and contributes to the possibility that "recurring disputes [will] fester outside the negotiation process until strikes or other forms of economic warfare occur." [The quote is from an earlier Supreme Court labor-related decision.]

In other words, collective bargaining should be in everyone's best interests. I believe that this fact holds for graduate student worker unionization, for non-tenure-track faculty unionization, for tenure-track faculty unionization, and indeed for all forms of unionization within colleges and universities. Collective bargaining is not only a way to protect individual employment rights. It is also a way to make institutions as a whole better.

It is also worth keeping in mind that organizing and collective planning can yield significant results even before they result in any kind of official recognition. At Chicago, for example, GSU has managed to do a great deal for its members, including influence institutional policies (despite our administration's refusal to even name them in communications), as well as for other organizations and causes through its solidarity actions and shows of support. Sometimes it can even be just a matter of being able to reach a large group of people with a common cause, or being able to bring enough people together to share information and counter strategies aimed at siloing different parts of an institution, or different categories of workers (as well as students and community members). Legal recognition is fundamental, as a tool and as a motivating goal, but it is not a precondition for a great deal of effectiveness, nor do we have to agree completely on the details of what we are building towards. Organizing is powerful in itself. Working things out collectively is powerful in itself.

None of this is to say that it won't require great effort to change institutional and individual minds. For reasons I will discuss below, I believe it is an effort well worth making, but it will be a struggle, no doubt. Many institutions, my own included, have fought academic unionization, especially for graduate student workers, in ways that have alienated even some faculty members who oppose it. At the same time, many other faculty members have bought the administration line entirely, even some who otherwise consider themselves aligned with the labor movement and other progressive causes. In addition to the issue of how we see ourselves and our work, there is also the effect of the entirely one-sided picture that administrations so often paint, using their control of the means of communication to do things like pass off rehashes of rehashes of texts prepared by lawyers and public relations specialists as the carefully considered personal opinions of deans, provosts, and presidents. It takes some work to learn enough to parse this kind of propaganda, but I think it is necessary work for anyone who wants to be a responsible academic citizen.

Of course there are financial considerations at play in this kind of opposition by administrations, and most definitely the desire to hoard decision-making power, whether

for personal gain or out of a genuine belief that "daddy knows best". But there is also an ideological consonance on the part of many academic administrations with the class that keeps the lights on. (How much of that is appeasement and how much genuine agreement is usually an open question.) And some of these administrations' responses have been so unnecessarily heavy-handed, so out of keeping with their usual political and PR savvy, that I have come to believe that not only do they want to fight organized labor, they want to be *seen* to fight it, by boards of trustees and other donor-class interests.[12]

Which brings me to the crucial context for this discussion, the contemporary academic environment that neoliberalism has wrought. Academic corporatization is an umbrella term that stands in for a variety of multifaceted but interconnected phenomena (some of which were apparently already clear to Justice Brennan when writing his Yeshiva dissent in 1980): the ascendancy of free-market thinking and human capital theory within academia, the influence of large donors, the rise in metrics and rankings, the move away from the humanities and towards areas like "big science", the bastardization of the struggle for equity and social justice into the corporate weaselspeak of so many "diversity and inclusion" programs, the view of students as customers, and so on.[13] It is a problematic term, not least because it often comes with a counterproductive nostalgia for a time that never was, a point I'll return to later in connection with [**6**]. But the term will do for this discussion. (By all means look for more nuanced views in recent work such as [**5**].) Especially if we keep in mind that this is a two-way street: academia is not just subject to these phenomena; it is a major generating force for them, both through the direct actions of institutions with substantial local, and in some cases national and global presence, and through its roles in ideological creation and reproduction in both scholarship and pedagogy, and in subject formation. (I am, after all, writing from the home of the Chicago Boys and their heirs.)[14] In this context, mathematicians and mathematics departments are in an interesting position.

Organizationally, the size and structure of many of our departments depend heavily on "service teaching". There are serious questions about the long-term sustainability of this model, especially as turf wars encouraged by a competitive, free-market approach and "butts in seats" budgeting encourage other departments to teach their own basic math courses. There is also the question of how to oppose the increased use and exploitation of adjunct faculty, if service teaching is the justification for our departments to be as they are. And the question of how to justify and defend tenure—one of the few legally-protected bulwarks against administrative overreach that some of us still have—if it is not being used as an instrument of solidarity, and is instead increasingly becoming part of an unsustainable divide between academic haves and have-nots that threatens ultimately to swallow even the haves (see for instance [**8**], and many of

[12] See [**15**] for a discussion of both tactics and ideological motivations, in the particular but generalizable case of graduate student workers and the University of Chicago.

[13] Though that last one, disturbing as it is, seems almost quaint to me by now. I feel that many institutions no longer see students even as customers, but as product. The donors are the customers.

[14] It is also important to note, that, through their tax-exempt status and extensive use of public resources, even so-called private academic institutions cannot be thought of as private entities with no inherent responsibility to the public good, a point well made by Destin Jenkins in [**30**]. Their exclusionary practices are thus often a form of expropriation. When they serve the interests of corporations and large donors, they then engage in a form of double expropriation, allowing what should belong to the public to be bought by those whose capital is itself generally the result of the privatization of public resources.

the articles linked to in that piece).[15] Intellectual and moral arguments against adjunctification, and for the protection of tenure and other safeguards against exploitation, might be doomed to failure at many institutions. These are labor issues, a couple among several in which we should seriously consider the benefits of being able to demand rather than plead.

Intellectually, I think there are several ways in which mathematicians share the position of the humanities, at least to the extent that we retain a commitment to work that falls under the rubric of "basic science". In this regard, the situation of the humanities might be a canary in the coalmine for us. When Kevin Birmingham became the first adjunct faculty member to win the Truman Capote Award for Literary Criticism, he gave an acceptance talk [**4**] in which he called out his tenured and tenure-track colleagues for complicity in the adjunctification of the humanities, arguing that it relies on the overproduction of PhD students. He said in that talk that "[t]he key feature of adjunctification is a form of labor-market polarization. The desirability of elite faculty positions doesn't just correlate with worsening adjunct conditions; it helps create the worsening conditions", and later that "[i]f you are a tenured (or tenure-track) faculty member teaching in a humanities department with Ph.D. candidates, you are both the instrument and the direct beneficiary of exploitation." (See also [**1**].) This is an important warning. Being in a field that might not be in quite such dire straits yet, and one with greater "marketability" outside the academy, should not be a cause for complacency. Indeed, the possibility that we might still be able to do something about it makes it all that more important to listen to such warnings, and to fight now for labor structures that will not pit us against each other in a desperate fight for resources made to seem scarcer than they really are by their poor distribution.

In the mid-1990s, the University of Rochester announced its "Rochester Renaissance Plan" [**35**], which among other things included a massive reduction in the number of math faculty and the elimination of the math graduate program. I was a graduate student at the time, and remember being heartened by a swift response not only by mathematicians but also by academics in other areas [**13**], who helped pressure the university into dropping that idea [**36**]. This kind of top-down restructuring continues to be a threat, and the economic climate as I write makes it likely that it will become increasingly common. Of course, when some institutions are being forced to close their doors entirely, "downsizing" starts to look like a reasonable compromise by comparison. But restructuring plans that are not made in collaboration with those who know the prevailing conditions on the ground, and will be the most directly affected, are almost necessarily doomed to be dismal reflections of blinkered thought, flexible morality, and institutional magical thinking. And given the interests at play, real collaboration needs at least a modicum of power balance.

For personal reasons, I followed some of the developments in a recent exercise in large-scale restructuring, this one at Goucher College, which closed several majors [**9**],

[15] If the protection of academic freedom and freedom of speech is a major argument for tenure, then it seems that those of us with tenure have a duty to help defend these freedoms for those who do not enjoy the same protection. This cannot be done via the kind of formal equality associated for instance with the "Chicago Principles" touted by the University of Chicago's president (and fellow mathematician) Robert Zimmer, which completely ignores differences in power and access, and can at best produce a world in which all can speak but only those in power can be heard. In this way, unionization is very much a free speech issue.

including mathematics. I couldn't help thinking as I saw some of that unfold that, with the passing of a couple of decades and the different situation and visibility of a liberal arts college, responses like that in the Rochester case are likely to come only if there is not just an internal organization of academic labor, but cross-institutional instruments of solidarity that can significantly mobilize faculty at other institutions, including more nationally visible ones. Similar thoughts about the possibilities of solidarity occurred to me locally when Chicago State University went through a financial crisis, which included all 900 of its employees receiving layoff notices in 2016, closely followed by its president leaving with a $600,000 severance package [**20**]. And of course more recently, as many institutions slide into insolvency largely unmourned by wealthier ones that can see the world only in competitive terms. But it is a competition with few real winners other than the large donors who receive the return of prestige and influence for their investments, and the administrators who use parameters such as that excrescence, the U.S. News & World Report rankings, to further their careers.

On the other hand, a dissociation from the most threatened parts of the academy, and an increased alignment with applied science and engineering, carry their own questions, raised for instance by funding sources, political issues like those we have seen around climate science, and the labor arrangements of labs. Collective action is key to protecting ourselves to the extent that we remain committed to work that is not at the center of corporate interests, and to protecting our souls, so to speak, to the extent that we don't. The structures of organized labor are not the only forms of collective action we need to engage in, but they can form a major part of that effort, even its center if done on a large enough scale.

At the same time, in some institutions at least, mathematicians might be in a reasonable position to resist some of the more noxious effects of corporatization, and hence might well have a responsibility to do so, because we are not as dependent on precarious labor and outside funding as lab scientists, or as under duress as those in the humanities. Perhaps there is a kind of rough analogy that can be drawn between academic corporatization and gentrification. In both cases one could use a positive phrase like "economic development", and certainly we shouldn't disregard the significant marshaling of resources that absolutely crucial areas of research and teaching such as climate science require.[16] One can make similar arguments about the needs of cities, but development *can* be done carefully and thoughtfully, and in particular it can be done with explicit, legally-enforced power-sharing, via instruments such as rent control and community benefits agreements. Indeed, the real engagement with community needs represented by such instruments seems to me a precondition to avoiding disaster. Labor unions can fulfill the same kind of legally-protected power-sharing role in the academy. In this analogy, mathematics departments and mathematicians might be a bit like higher-income members of communities undergoing gentrification. We see ourselves as neither gentrifiers nor gentrified against, for now. Then some questions arise: What are the responsibilities of those who are not being displaced to those

[16] On what it will take for academic institutions to respond in a responsible manner to climate change, including the role of unions and coordinated action between them, see Izabella Laba's clear-eyed and moving contribution to this volume, posted online at [**17**].

who are? How sure are they that they will remain in that position, and what compromises will that involve? Even if all goes well, will the resulting environment be one they want to live in?

But why not pursue collective action through existing means, formal ones like organs of faculty governance, or informal ones like lobbying through department chairs, ad hoc meetings with administrators, petitions, and so on? And for those for whom recognized academic unions are already an existing means, like faculty at some public institutions, why put in the effort to strengthen them, and make them part of a broader, intra- and interinstitutional solidarity movement? The moment we find ourselves in as I write might have something to teach us here.

A lesson of the pandemic: Things are fine until they're not, $n^2 + n + 41$ is always prime until it isn't, and the cost of not being prepared can be staggering. This is true at the individual level, and all the more so at the communal one. Asking why someone without major grievances would want to unionize is like asking why a healthy person would want to get health insurance. But no, it isn't. It's like asking why a healthy person would want to have a robust public health system.

A second lesson of the pandemic: When things go wrong, current institutional structures will not save us. The extent to which faculty governance has eroded at most institutions has been placed in stark relief by the lack of democratic faculty participation in planning for teaching and research during this emergency. I'm not talking here about having some faculty presence in committees with no ultimate decision power. I'm talking about the call made in the AAUP's "Principles of Academic Governance during the COVID-19 Pandemic" [**33**] to "honor the faculty's decision-making responsibility for academic and faculty personnel matters as the most effective means of weathering the current crisis", and the reminder that, in well-functioning institutions, "[n]o important institutional decision should be made unilaterally by administrations or governing boards." At the University of Chicago, for example, as I write the elected Council of the University Senate, which according to the university's statutes [**34**] is "the supreme academic body of the University" has not been called to vote on any aspect of the teaching model to be adopted in the Autumn of 2020. I am well aware that at other institutions, faculty participation in decision-making on this issue has been even less than at Chicago. It should be clear that, especially when it comes to matters where the financial interest is likely to outweigh the academic one, we cannot rely on informal influence, nor unfortunately on formal organs of faculty governance, as much as I am still willing to fight for this possibly outdated notion,[17] for similar forms of institutional democratization, and for cross-institutional initiatives like the COVID-19 Statement of Academic Solidarity [**27**]. To have a voice, we need real power. We need union power.[18]

[17]Some of what I say about the empty shell that faculty governance has become at many institutions comes from the experience of having served a couple of stints on the Council of the University Senate.

[18]Sara Matthiesen's "How to Stop the Cuts" [**18**] has several examples of recent collective action by academics that has had real positive effects. See also [**24**] for an extended discussion of the case of Rutgers. I can add the example of the University of Chicago's non-tenure track faculty union, which negotiated a contract with significant improvements to its members' working conditions in 2018. The negotiations were long, and involved the possibility of a strike. Our AAUP chapter wrote a public letter in support of these colleagues as they prepared to make the exceptionally difficult decision of authorizing a strike, and organized within departments to obtain over 200 tenured and tenure-track faculty signatures. This effort was nothing compared to the enormous work put in by the union's negotiating team and other organizers, but an important

And so a third lesson of the pandemic: To riff on Churchill, collective solutions are the worst solutions to our problems except for all of those others that have been tried. Organized collective action should not be entered into thoughtlessly. As much as we might decry those who fail to participate, we have to keep in mind that there is often a solid history behind their distrust of the collective. The purpose of unions is to check power with power, so if they are working correctly, they will be powerful. And power doesn't only corrupt, it corrodes. The work to keep this from happening is endless, but the alternative is this great perversion of the idea of individual freedom that animates the maskless libertarians of the current moment, whose actual freedom extends exactly as far as those who wield real power want it to. That their horror of being told what to do has a history does not mean that it isn't hugely self-defeating, as well of course as all-of-us-defeating. If the day ever comes that we have a full faculty union at my university, I will celebrate that evening, and show up the next day ready to fight to make sure the union is and remains democratic, transparent, accountable, and committed to the greater good. *Hay gobierno? Soy Contra!* But opposition can still be constructive when the balance of power has not shifted so far as to be not only out of reach, but even out of sight. In a union I would have a vote, but I think I would have more luck finding out the formula for Coca-Cola than just trying to learn what goes on at the meetings of my university's Board of Trustees, let alone affecting any of their decisions.

As wary as I am of the rhetorical trick wherein one presents a caricature of others' arguments and then proceeds to destroy them like an action hero in a choreographed fight, I think there is value in going through some other possible responses to corporatization, in very broad strokes, and responding to them in turn.

One, of course, is to see it as generally a good thing, to think that the academy needs to be more like the business world anyway, let the free market work its magic, etc. That world-view might be a little too far from my own for me to engage without writing a whole other essay, but I'll say that it's a miserable tautology to assume that a system is working because it does well by those who "deserve it most", where the latter are defined exactly as those who have become successful under the system. (A particularly salient point for a field like mathematics that is so implicated in the reproduction of the myth of meritocracy.) Brazilian writer Luis Fernando Veríssimo has an essay [**22**] in which, speaking about neoliberalism, he tells the story of a particular kind of dolphin that has become well-known for saving people from drowning by pushing them towards the shore. There is a theory, he says, that these dolphins just like to push people. Some of them get pushed towards the shore, some away from it. But no one gets to hear the version of the drowned.

There are also those who deny, explicitly or implicitly, that these things are happening, at least at their own institutions. If that is really the case, then congratulations. But even then I would ask: What safeguards are in place to prevent it from happening in the future? What mechanisms of solidarity with other institutions are being established? It's not exactly wise to say everything is fine because it's just my next-door neighbor's house that is on fire.

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aspect of solidarity is that every little bit helps, and focusing mainly on showy, heroic actions is ultimately counterproductive.

Another response is that, whatever we think of it, corporatization is an inevitability that we need to learn to work within rather than try to resist. As mentioned above, probably the most common version of this response is implicit, through keeping one's nose down and not worrying about anything beyond one's own work. Similar in effect is the position that things are too far gone for anything to make a difference. I think that much of what I'm saying in this essay is that there is still much that we can do, and that fighting for changes, however small they might seem individually, is both an obligation and an exercise in self-interest. Especially because whether or not things can get better, they can definitely get worse. (See for instance [**2**].)

Even the most cynical versions of this perspective tend to come with moral self-justifications and all their typical attending contradictions. I found it rather interesting to hear of an account from someone in the lab sciences of some of their colleagues' reasons for opposing a graduate student worker union, which included both the belief that graduate students in the lab sciences are richly rewarded, so that they have no grounds for complaint, and the knowledge that many lab leaders are asking too much of their students and should be worried about external oversight.[19] (But of course, it's all in the name of science…)

I have much greater sympathy for those who choose to operate within the existing framework by using informal power channels to work on others' behalf, or in the collective interest. (I'll say something about more formal means below.) To some extent, this is necessary work even within highly functional organizations, given that a perfectly flat power structure is something of a point at infinity. An overreliance on it is dangerous, however. For example, I suspect that there are many politically savvy departments that have found ways to improve the conditions of their graduate students beyond those of others at the same institutions through backroom negotiations. But I am not ready to give up on democratic institutions and transparency, or accept the distortions and inequities that this informal approach creates. Fostering internal competition for resources, some of which have been made artificially scarce, is a key strategy of the corporate power structure. And it's dangerous to feed the monster in the hope that it will never turn against us. That's just not how monsters work.

Nor should we acquiesce to the paternalism of administrators who demand our trust that they have our best interests at heart, and quickly become offended when we question the wisdom of blind trust. It is often difficult to read this offense as anything other than strategy, when they must know full well that, by giving up our right to share in the decision-making, we are extending our trust not only to them, but to all of their as-yet-unknown successors.

There are also those who see taking care of oneself or one's immediate community as a form of resistance. I do have some sympathy for this idea too. One version of it can be found in Maggie Berg and Barbara K. Seeber's *The Slow Professor* [**3**], which advocates adopting principles of the Slow movement (probably most famous in its Slow Food form) as a form of resistance to the demands for increased speed and efficiency in the academy. Although there is much in that book I agree with, I was struck by

[19]The idea, so beloved of private university administrators, that unions will drive wedges between graduate students and their advisors is generally drivel. If anything, a graduate student worker union would be a welcome ally in advocating for my students in areas over which I have no direct influence. I will say, however, that there are advisors out there who are right to be worried about the increased protections against exploitation and harassment that unions can bring. These, of course, are advisors we would be well rid of.

the lack of discussion of demands on our time and energy placed not by corporatized interests themselves, but by the need to stand with those who are most affected by these interests. This is also labor that tends to fall disproportionately on women and people of color, all the more so in a field like math with its deep gender and racial imbalances. The kind of slowdown through collective and institutional action advocated by Izabella Laba in [**17**] seems to me a much better version, and as noted near the end of that piece, unions have an important role to play here.

I see a similar problem in the way that the influential idea of being "in but not of" an institution, from the chapter on "The University and the Undercommons" in Stefano Harney and Fred Moten's *The Undercommons: Fugitive Planning & Black Study* [**11**], to "disappear[ ] into the underground" of the university, can be taken as permission to check out. I think of the work I do in criticizing, protesting, and organizing against some of the policies of the University of Chicago's administration as part of my service to the university. I mean that quite unironically. It might not be serving "the University" if we define it as coterminous with its official version, or its current policies and practices, but this work is meant to serve a different university, a potential one that I am happy to be of, not just in.

Even the more "traditional" academic service I do, sitting on various committees and such, is something I try to do with that university in mind. I should make sure to reinforce here that I think of responsible, thoughtful work undertaken as a committee member, department chair, or even a higher-level administrator as potentially a "yes and" thing. It is definitely true that much of this work helps to buttress problematic structures, but that is one major reason that I want it to happen in a context in which there are other structures in place to help fix these problems. So, for instance, I will serve on my department's graduate admissions committee regardless, but I would be happier doing so if I knew that our incoming students were entering a university where they could be part of a recognized union with the ability to negotiate with the university administration in ways that I, or even my department as a whole, cannot, for example when it come to issues related to health benefits. I would be even happier if the academic labor system many of them will continue to inhabit after graduating were a more humane and equitable one, protected by real power-sharing and solidarity among democratized institutions. And while this kind of service work can certainly help if it is done with the right goals in mind (which is not possible for all positions, unfortunately), it is not enough, for similar structural reasons that make organs of faculty governance not enough.

A final response I want to consider is the one that holds that the way to resist corporatization is to resist everything that comes with it, including reactions to it like unionization. I have talked to academics who see efforts at academic unionization as something that creates, rather than responds to, the transformation of education and research into a business. All I can say there is that I think that the corporatization ship has long since sailed, and that this is exactly the wrong way around. Can one really look an adjunct trying to cobble together a living out of several part-time positions with no benefits in the eye and say that it is their desire to improve academic labor structures that is sullying the beautifully pure and abstract world of the ivory tower? Or can one deny the relevance of that adjunct's working conditions because they are not working in one's particular institution or department (yet)?

This perspective reminds me of the situation in which those who want to address issues such as racism, sexism, homophobia, and transphobia in the arts, or the academy, or any number of other settings are accused of wanting to "politicize" things that were somehow beautiful and pure and blessedly unworldly. This kind of argument might be out of favor at the moment, as individuals and organizations are forced to confront one of the many truths that the Me Too Movement and Black Lives Matter have put in front of us, that the "neutral" position is one of the most politically-charged of all. But it will return, and it will find new places to deploy itself.

A particular feature of this last perspective is a preservationism based on the image of a past in which the academy was a highly functional community of scholars. There are others who, while believing that there is little or nothing left of this past to preserve, nevertheless see a return to it as the ideal towards which our reformist work should be directed. But those who would live in Xanadu always imagine themselves the Khan, never a servant. I hope it is no longer controversial to point out the general idea that the "good old days" were good for very few. Even if there are aspects of how colleges and universities used to function that we would like to recapture, a naive regress, even if it were possible, would get us nowhere near where we need to be. We need a way forward that minimizes the harm done by our individual inabilities to see much beyond our own interests. To me, a promising direction is pointed to by broadly collective organizations like the UChicago Labor Council, which brings together officers, stewards, and members of various university and university-related labor unions, academic and nonacademic alike, as well as other solidary groups, including student and community organizations. The aims of the council include mutual support, knowledge-sharing, the coordination of efforts, and visibility. Particularly important is its underlying premise that there is common cause between workers of various kinds at the university and beyond, as well as those affected by its policies.

I would also like to highlight the case of Rutgers University, as outlined by Todd Wolfson, president of the Rutgers AAUP-AFT Executive Council, in his recent interview [**24**] with Astra Taylor, where he describes an expansive and ambitious kind of coalition-building, and an organizing framework known as Bargaining for the Common Good [**26**]. I am very glad that this interview was published before I had completed revisions to this essay, so that I could cite it and note that Wolfson discusses many of the issues I've touched on here, with the concrete details of a particular multicampus institution's realities, making a compelling case for a broad form of solidarity within the context of academic labor organizing.

In "Critical University Studies and the Crisis Consensus" [**6**], Abigail Boggs and Nick Mitchell point out that critics of academia of all political stripes agree that academia is in crisis (and this pre-pandemic…). They say that

> [t]he crisis consensus is a mainstay of political ideology that functions with particular ardor in higher education, where it pivots on the invocation of the university as a good in itself, as an institution defined ultimately by the progressive nature at its core. The crisis consensus thereby settles in advance the constitutive problems and paradoxes—to say nothing of the forms of real expropriation and violence—that continue to constitute the university as such.

Later, talking about the field of Critical University Studies specifically, they further point out that this work has too often made the crisis consensus normative, and that this consensus itself is "normed, often silently, by an analytical predisposition toward rescue and restoration."

There are a couple of vague thoughts I have had a for a while, which crystallized when I read their analysis. One is that this predisposition is particularly relevant to unionization because the "good old days" that many critics of present-day academic institutions long for did not have academic unions as an important constituent part, or to the extent that they did, these unions' contributions have been largely forgotten. Unionization as a way forward may not appeal to those who would rather move back in time. Even with the best intentions, the desire to revisit a mythical past (which in reality was worse than the current moment in several aspects) can serve to bolster the status quo by undermining institutions like academic labor unions, especially in their role within a new labor movement.

The other thing that particularly struck me while reading [**6**] is just how clearly right its perspective appears to be to the people I interact with in settings like the Labor Council and other organizations that involve people whose relationships to the university are very different from that of tenured faculty. Community organizations in particular have the memory that whatever good old days there might have been within the academy were bought at a great cost to many outside it (and, of course, to many within it). To give just one example of how this process has worked, the Reparations at UChicago Working Group (see [**14**]) has built a case based not only on the original founding gift to the university from capital gained at the expense of enslaved people, but also on a long history of complicity with racist practices extending well beyond its early days. Of course, it's possible to separate an institution's internal and external practices, faculty can be teaching responsibly even when their institution behaves irresponsibly, camels do sometimes pass through the eyes of needles, but it's an iffy proposition. Particularly so for those who devote significant labor to institution-building, who then take on a responsibility for the actions of the institutions they help build, and deserve a real say in those actions, even outside their particular spheres. It might be a strange idea to some that helping to build up a mathematics program, say, confers both a duty and a right to play a part in determining the overall practices of the institution within which it exists, but it should be a powerfully motivating one if we are willing to engage with it seriously.

One doesn't have to agree with the specific diagnoses or demands of organizations that bring a critical eye to the practices of colleges and universities to find their perspectives important,[20] to see the benefits of having a broader array of voices at the table, or to want to work toward a more democratic future, indeed, one in which we ourselves can have a greater say. It does require us to talk to each other, to negotiate alliances, and not to think of our institutions as the only sources of possible solutions. It also requires us to educate ourselves about organizing strategies, to understand processes

[20] In fact, these organizations often disagree with each other, which is of course very healthy. When these disagreements happen, however, universities often seek those whose positions are most closely aligned with institutional interests, and market their voices as the predominant "will of the people". My point here is not that university administrations are unique in amplifying only the voices they agree with, but that on many issues, they are currently the only ones with a sufficient loud amplifier to be heard above the ambient noise.

like power mapping (see [**18**]), perhaps to put into real practice what is sometimes an empty commonplace and learn from our students. And it does raise the distinct possibility that we ourselves will be called to account for some of our own practices, will be made uncomfortable, and will not be able to remain in our carefully carved-out niches in the status quo. None of these are drawbacks.

Some of what I've said above might sound dismal and depressing. I know there are those who see as dismal in itself any analysis of academic life that describes it as at least in part a system of power relationships. To me, it's only the same kind of acknowledgment of reality that architects and engineers must face when building on difficult terrain. I began this essay with a sense of possibility, and I want to end that way as well. It's not easy to keep that spirit up, particularly so at this time in which I'm writing, but I don't think we have the option to give up. It's once again time to do the Hokey Pokey while walking in a circle.

Labor unions are a tool, not an end in themselves. They are not the only collectivities we need to build on and around campuses, and they certainly should not be a way for us to abdicate our responsibilities to an organization. As I said before, unions have suffered from many corrupting and corroding forces, and a 21st century labor movement requires constant participation and vigilance, in all directions. We need to ensure that unions work in our interest and that of the common good, and we need to fight institutional reactions, including ones deliberately designed to cause damage and blame it on unions. For instance, it is a central part of universities' union-busting playbook to try to portray graduate student worker unions and non-tenure-track faculty unions as oppositional to tenured faculty. It is particularly important to resist this trope, which is part of the overall corporate strategy of turning workers against each other.

So yes, labor unions are a tool, but they are a crucial one. We need institutions with real power, to demand and decide, not merely to request and advise, and to help set the agenda in the first place, so that our decisions are not merely symbolic. We also need collective resources and organizational infrastructure. It is very difficult to conduct productive negotiations when one set of participants has an array of resources, including the services of dedicated professionals, while another has to rely entirely on a small group of volunteers whose energy might disappear at any moment. The floors of the academy are littered with noble causes dropped as soon as the two or three people leading the charge found themselves faced with too many other, more pressing demands. Meanwhile, an institution's lawyers or PR staff will never give up on an issue because classes have started and they have a pile of homework to grade.

More than anything, though, we need solidarity. This is not an empty word; it's a real, day-by-day, lived thing. It's a matter of showing up, of doing the work, of supporting each other, of organizing and re-organizing and negotiating, but never losing track of what we can build together. Of believing, for example, that through collective action we can flatten the income distribution of academic and nonacademic workers within and across institutions, and that doing so will produce a better environment for all of us. It's a matter of using whatever power we might have, and of joining forces to amplify this power. As Carolyn Betensky of Tenure for the Common Good says (as quoted in [**19**]): "This isn't a savior thing." This coming together of interests is powerfully summed up in the words, sometimes attributed to Lila Watson but according to her, more properly credited collectively to an Aboriginal activists group she was part

of in 1970s Queensland: "If you have come here to help me, you are wasting your time. But if you have come because your liberation is bound up with mine, then let us work together."

When I think of academic unionization, I don't think only of individual unions negotiating over local employment conditions, as important as that is. I think of an opportunity for collaboration and solidarity within and across institutions on a range of issues, involving many moving parts, with unions forming a key component, even if not the only one. Yes, salaries, timely payment, benefits, grievance procedures, health care, but also shared governance, class sizes, student loan burdens, responses to climate change, helping those at financially-strapped institutions through cooperation, and very much also the responsibilities of academic institutions to individuals and communities, their political roles at various levels, and how we can act as a powerful force within a new kind of labor movement.

So my call to action is not a single-minded one. I very much believe that we should be organizing to create new unions and get them recognized; strengthening existing academic unions, and fighting to keep them focused on the broad struggle for equity and justice on all fronts; and building connections between unions, academic and nonacademic alike, as well as between them and other organizations within and outside academic institutions. None of this requires heroic individual effort. Each of us can do our own bit of the work and encourage, help, and when necessary and possible, protect others who are also willing to do theirs. We can also take small collective actions without a full-time commitment to a cause. Starting a chapter of the AAUP, for instance, is not difficult. But I also believe in the importance of an individual and collective rethinking on the part of academics of the structure of our institutions, their internal and external impact, and our own status as workers who share a collective interest that goes well beyond their carefully guarded walls. From this kind of understanding, the work can flow naturally, as naturally as the things big and small so many of us do every day within our mathematical communities flow from our love of mathematics. (And yes, there is nothing wrong with love, when it doesn't blind us, or bind us to an unjust system.)

It's an old saw, but its teeth are still sharp: "United we bargain, divided we beg." I am under no illusions that sustained, well-informed collective action, even unto a fully unionized university, can solve all the problems of the contemporary academy. But it is our best hope to make at least a turn toward more democratic, more just, more humane institutions, ones in which we can thrive ourselves, without being forced by the need to keep our heads barely above the water into practices and alignments that ultimately hurt us, our students, our fellow workers, and our communities.

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Universities in the time of climate change

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1. My first attempt at this essay grew out of my frustration with common institutional responses to the climate emergency. "Sustainability" has become yet another bonanza for developers and manufacturers. New energy-efficient buildings are joyfully constructed, appliances are replaced as soon as a newer and slightly more efficient model becomes available. A typical sustainability webpage boasts of new construction, fundraising, and multimillion "green" developments, with a sprinkling of low impact feel-good projects on the side: bikes, straws, reusable coffee mugs. Institutions act as if "shop more, save more" were deep words of wisdom that applied to the environment, as if we could address a crisis of uncontrolled expansion by doing more of the same. As for the employees and customers, or faculty and students, we are expected to allow ourselves extra time for construction-related detours on our way to work, yield the right of way to heavy machinery, take a yoga class if we discover that we have anger management issues, and otherwise continue as usual.

I spoke about this, remotely, at the JHU Workshop on Professional Norms in Mathematics in September 2019. I wrote in my set of slides for the talk:

> Climate change will be hard on us, both physically and mentally.
> Heat waves, wildfires, air quality, disaster preparedness and responses,
> power outages, boiled water advisories, etc.: we will not be able to
> rely consistently on modern age conveniences.

When the slides were circulated on Twitter and blogged at the Azimuth [**B**], reactions were divided. One tech person said that this was nonsense: we would be able to shield and air-condition a university in the middle of the Death Valley if needed, this would be an obvious priority given that the future of humanity depends on the continued ability of the smartest people to work in comfort. A few weeks later, under the

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threat of wildfires, the California utility PG&E cut off electricity to various locations including the Berkeley campus of the University of California.

I also wrote this:

> We will not be able to demand that everyone must operate at 100% capacity, 100% of the time. Employers will have to acknowledge that people are human, and plan accordingly. If lack of resources does not stop us, public health issues will do it.

I did not know that a global pandemic was just around the corner.

Universities have not acquitted themselves well in the Covid-19 crisis. In the most benign cases, faculty scrambled to set up instant online courses using whatever resources they happened to have at home, while administrators wrote fanfics about their endless emails, memos and directives providing support and leadership during that difficult time. Furloughs, layoffs and restructuring have already started at many institutions. Students are often left in a limbo, the future of their education uncertain, their housing and immigration situations hanging by a thread. The worst is likely yet to come, as too many universities still plan to reopen in a manner that will put the health and lives of their students, faculty, staff, and surrounding community in grave danger.

Yet, this is only a small preview of what climate change will bring. Other pandemics may follow, as we continue to encroach on parts of the biosphere that would be best left to themselves. As wildfires ravaged Australia a few months ago, we watched helplessly the cell phone videos from residents who, having been told that it was too late to leave, sought shelter by wading into the ocean instead. Extreme conditions will become commonplace. There will be no help coming and no one available to bail us out when everyone's resources are strained to the limit.

We should not want to return to the "normal" from February 2020. That was not sustainable even before the crisis hit. The campus I work on was too large and too crowded, its layout in open conflict with class schedules and student timetables. We were rushing people from place to place with not enough time to get there, packing them into classrooms that were too small, figuring out whether to close the windows so that we could be heard against the noise of the leaf blowers or keep them open so that students do not suffocate. The sheer logistics of all that took so much exertion that little was left for the actual education part.

Expansion comes at the cost of resilience. Campuses were already coming apart at the seams. They had trouble accommodating lesser crises, from snowstorms and transit strikes to the H1N1 threat a few years back. Their only safety valve has been to ask students and faculty to go above and beyond, sacrificing their personal time and resources to maintain the illusion of the institution working as usual. Of course this had to crumble in a global crisis, and will continue to crumble as challenges keep coming.

We could continue in this manner, asking people to perform heroic feats so that institutions could pretend that everything is fine. Or we could acknowledge the reality. We could slow down, scale back, allow room for buffer in case of emergencies or unexpected circumstances.

I do not have confidence that universities will make the right choice.

FIGURE 13. View from Rebecca Spit, Quadra Island, BC

2. This is a personal reflection. I am neither a climate scientist nor a fortune teller. I did, however, grow up in a country where scarcity was the norm and adaptation to difficult conditions was a daily fact of life. Poland, devastated by World War II just a few decades earlier, then oppressed and exploited by the Soviet Union as it tried to rebuild, had low living standards across the board. Sustainability was not a buzzword and we did not have corporate offices dedicated to it. What we had was very limited resources.

The centrally planned state economy was wasteful, inefficient, and inflexible, doomed by its disregard for expertise and inattention to local specifics. On the level of individual households, we lived frugally, trying to do the best we could with what we had. Reuse, repurposing, and repair were not a boutique option, but an economic necessity when new consumer goods were scarce or unavailable altogether. Contingency planning was essential. Power and water outages were common and could happen anytime, and the supply of basic food and hygiene products was never guaranteed. We could neither outsource our problems nor buy our way out of them.

Here and now, the issue is not just that we should learn to reuse and repair, although we should indeed do that. We also have to recognize that sustainability is labour-intensive, requires constant human attention, and might not drive economic growth as measured by capitalism's preferred indicators. As our circumstances become more difficult, we must anticipate the human impact of widespread and prolonged hardship. Many in the West are getting their first taste of it as they find themselves less productive during Covid lockdowns. This is old news to those of us coming from countries that have seen harder times.

I also come from a society that could not afford to take its science, culture, or intellectual life for granted. Throughout the 19th century, Poland, partitioned between Russia, Prussia, and Austria, did not exist as an independent country. Teaching Polish history and literature was not permitted under Russian rule, so Poles organized their

own illegal schools. Marie Curie, born Maria Skłodowska, attended the secret "Floating University" in Warsaw which changed locations frequently to avoid being detected by the Russians [**D**].

FIGURE 14. The house where Marie Skłodowska-Curie was born, Warsaw

Underground classes returned during the German occupation of Poland in World War II. The Nazi plan for Jews is known well enough; Slavic Poles, just one step up in the racial hierarchy, were to become slaves working for the German empire. Hitler understood, as many dictators do, that nations are easier to subjugate when their intellectual elites are eliminated. About 20% of Poland's population were killed in the war; among those with higher education, the fatality rate is estimated at 30-40%. The Jagiellonian University professors were deported to the Sachsenhausen concentration camp as part of the *Intelligenzaktion* in 1939-40. By the time the Germans took the territories initially occupied by the Soviets, they did not bother with the preliminaries. A mass execution of Lwów professors, Jewish and gentile alike, took place on the morning of July 4, 1941. Others were killed in the days and months that followed.

Stefan Banach, one of the greatest mathematicians of the 20th century, survived as a lice feeder [**U**]. Rudolf Weigl, an Austrian-Polish doctor researching typhoid vaccines, saved a number of scientists by enrolling them as experimental subjects in his institute. The mathematicians would sit at one table in the lab and debate theorems and proofs to distract themselves from the pain and indignity as the lab lice fed on them.

This is not an inspirational story of overcoming adversity. Banach died of lung cancer in August 1945. The Polish mathematical school did not return to its pre-war greatness. As Urbański writes:

> In his journal, [Hugo] Steinhaus made a list of the mathematicians lost to Poland during the war. Fifty names, including Lvivians: Bartel, Łomnicki, Stożek, Ruziewicz, Auerbach, Chwistek, Saks, Schauder, Hetper, Kaczmarz, Herzberg. He also counted those who left Poland in time, such as Ulam and Kac. "Almost 70 percent of research mathematicians who were Polish or originally from Poland," he wrote.

This is all very hard to write, but I want to share it because of the utter lack of imagination displayed today by too many administrators, college presidents, and other presumptive leaders. Do they really think that moving all classes online for a year or two is the worst that can happen? What will they do when climate change kicks out their door? It is not surprising that the clearest vision and leadership has often come from HBCUs and community colleges [**S**], where hardship is the daily reality and not just a paragraph in an admissions essay.

University faculty come from many countries and backgrounds. I have only read about wars and heard about them from my elders; there are faculty who lived through them. We have witnessed, and worked through, mass upheavals and natural disasters. Do not keep your consultations limited to mysterious "stakeholders," especially not to those who have never experienced a greater calamity than their stock options taking a sharp turn downward. Talk to people who have lived through a crisis or several. How important was education to them? How far would they go for its sake? Where would they stop? What would they prioritize? We are already finding out which jobs are really essential and how many of them require a college degree. We may be about to learn more about the difference between education and credentialism. The reckoning will not stop there. Universities, by and large, are not prepared to face it.

3. Many university campuses, including the one where I work, are near-permanent construction sites. The construction is often tied to sponsors, donations, fundraising opportunities with strings attached that pull us in non-academic directions. Stadiums, alumni centers, administrative offices, and other similar objects are prioritized over classrooms and research space. Large retrofit projects mandate the same improvements everywhere across campus, needed or not. With the choice and scheduling of projects contingent on the vagaries of external funding, there is little local input as to emphasizing academically needed upgrades or coordinating different projects at the same location. Faculty and students feel like intruders in spaces where academic activities are clearly not prioritized.

Much of this is advertised as environmentally friendly. The new buildings will have solar panels, certified energy-efficient fixtures, grey water recycling. The distinction between reducing environmental impact and merely shifting it elsewhere is conveniently ignored. Infrastructure retrofits can reduce emissions on campus, but the new energy-efficient appliances have to be manufactured somewhere, often in countries we criticize for their high carbon footprint even as we continue placing orders with them.

One might also ask when a university really needs to have another stadium and how many alumni centers are actually necessary. Over the decades, universities have

tried to become company towns, sports and entertainment centers, and real estate operations. Far from the minimalist model of dorms, food halls and a bookstore, they have branched out into everything from luxury condos to health services to sports and recreational objects, not to mention the ubiquitous branded apparel and giftware.

The stadiums and the alumni centers are empty now. The branded sports apparel sold at the bookstore is collecting dust. The company town is all but boarded up. Aside from the university hospitals, the only part of the university that continued to function without interruption is the one that received little or no investment and that did not require the company town or the luxury infrastructure at all: the actual education, the faculty teaching from their kitchens or living rooms, the students learning in whatever spaces they have available.

In a crisis, one must choose one's priorities. Education, scholarship, and research are the reasons universities exist in the first place. There will always be a demand for that, even in a crisis, for as long as human civilization exists. Can we focus on how to meet that demand reliably and consistently in changing circumstances? Or must we continue to invest in administration and infrastructure that will likely become dead weight in the near future?

I would very much like to have a sustainability office that would subject all construction on campus to careful scrutiny, approve only those projects that are critical to the university's research and educational mission, and disallow all those whose main rationale is that the money happens to be available. I would love to have a sustainability initiative that would replace the ubiquitous American-style lawns on campus with local low-maintenance vegetation that does not require sprinklers, toxic pesticides, leaf blowers or lawnmowers. Ideally, this would be done in a gradual, user-friendly manner, without the massive roadblocks that keep us from getting to classes on time. I would love to see sustainability projects that reduce noise and disruption, making room for the sustainable, environmentally friendly academic activities of quiet study, reflection, and conversation.

4. We are inundated with calls to change our personal habits. We are told to ditch our cars, bike to work, avoid plastic bags and disposable food containers, sort our recyclables, turn down the thermostat and put on a sweater. The encouragement takes the form of posters, placards, broadcast emails, bike to work competitions, and expensive parking permits that nonetheless do not guarantee a parking space. It is assumed that faculty and students live close enough to campus and have a safe cycling route to it, that they have a bike, storage space for that bike (nonexistent in many city apartments), waterproof clothing and carrying bags in case of inclement weather. It also helps if someone else (usually, a wife) is available to pick up the kids from school, buy the groceries, and run any other errands that require a car. The participant is assumed to be in good enough health and have enough time and energy left at the end of the workday. All of these factors tend to be correlated with race, class, gender, ethnicity, and income.

Universities might do well to listen to Dr. Bonnie Henry, B.C.'s provincial health officer, under whose stewardship British Columbia suffered minimal Covid casualties so far with relatively low levels of social and economic restrictions. Her office has always emphasized that government-level public health measures, and the resources

to support them, must come before policing individual behaviour. The *New York Times* reports on her approach as follows (emphasis mine) [**P**]:

> It was while working for the World Health Organization tracing Ebola outbreaks in Uganda that Dr. Henry developed her ideas about how best to respond to public health emergencies. The keys to an effective quarantine, she came to understand, were communication and support, like food and medical follow-up, not punitive measures.
>
> "If you tell people what they need to do and why, *and give them the means to do it,* most people will do what you need," she said.

Faculty often report 50-60 hour work weeks with little or no vacations. Class sizes are increasing. Digitization, instead of reducing our administrative workload, has increased it by redirecting much of the work from staff to faculty as self-service. Tenured faculty already do research, supervise graduate students, write grant proposals, serve as journal editors and referees. We are also asked to learn innovative teaching methods, monitor and support student wellbeing, engage in public outreach, and participate in initiatives to promote diversity and inclusion. These are good things to do, but can one person really do it all? In the limited time we have? The percentage of contingent faculty is also increasing. Many of them have extremely high teaching loads and must commute between two or more campuses in order to make a living. They cannot do that on their bikes, especially if they have no safety net to provide health care or replacement income if they get clipped by a car.

Tired and overworked people do not have the time or capacity to accept additional challenges. They will drive to work, order takeout food for lunch or dinner even if it comes in Styrofoam containers, forget their reusable bags, throw garbage in the compost bin by mistake, generally waste resources that could otherwise be saved. Simply telling us to stop doing that will never be effective. There are reasons why we need cars and convenience food. Those reasons must be understood and addressed, and I do not see how this can be done without putting workload reductions and improved working conditions on the table.

This will only get worse as climate change continues to affect our physical and mental health. Here in Vancouver, smoke and air pollution from wildfires has become a regular occurrence in the summer. Heat waves, floods, hurricanes become more frequent. New diseases emerge, old ones threaten to return. We will be exposed to extreme conditions more and more often, and will have fewer ways to mitigate them.

It is not enough for us, individually, to try to reduce our own personal activities that carry high environmental cost. We have to stop requiring others to engage in such activities, both through legislation and through professional and institutional norms. This is not just faculty versus administration, either. We have to stop making impossible demands from our trainees and colleagues. Then perhaps they will remember to bring their reusable mug and ride their bike from time to time.

There are other lessons from Covid that we should learn. For example, as Dr. Aaron Carroll explains [**C**], it is more important for public health measures to be implemented widely and easily than to work perfectly in every single instance:

> Public health experts focus more on huge groups, not individuals. They don't need masks to work perfectly for everyone. They're thrilled

FIGURE 15. Vancouver, Summer 2017 (The Fifth Season)

> to see a smaller benefit in a larger population. And there are models showing that if masks are about 60 percent efficient, fewer than three-quarters of people would need to wear them to keep a disease like Covid-19 in check.
>
> Today we're in danger of making the same mistake with tests... We have to start accepting less accurate, widespread testing for groups. We have to stop muddling the messaging by focusing only on the most effective tests. With testing, just as with masks, more is sometimes better than perfect.

The same will apply to climate change. We will not be saved by experimental hyperefficient cars or residences for the wealthy few. We will need cheap solutions that will be available to most people, products that can be made easily and inexpensively, procedures and regulations that scale well throughout a society.

Universities will have a part to play. To be sure, climate scientists are already doing their best. Technologies required for adaptation and mitigation will likely rely on knowledge created at universities and research institutes. We are counting months to the Covid vaccines currently being developed and tested by scientists, and since we have no way of knowing what will hit us next, our best bet is to continue to invest in a broad, flexible research base that can be deployed in multiple directions.

And yet, that by itself will not be enough. It will also be necessary to understand people and societies, and to design our climate adaptation measures accordingly. We should part, once and for all, with any notion of defunding humanities. History, sociology and anthropology must guide us in using the tools we develop. And, if universities wish to be credible leaders in this, they should start by applying such lessons to their own institutional structures.

We will have to learn humility. High-tech molecular tests are important tools for managing Covid, but so are lowly cloth masks. We will likely need similar combinations of high tech, low tech, and common sense in mitigating the effects of climate change. Universities can help by examining, without prejudice or condescension, solutions already used in other countries and cultures. They can offer frameworks for recognizing, studying and teaching traditional and Indigenous knowledge. We will need all the expertise and wisdom we can muster.

5. What would a better, more flexible, more resilient university look like? We certainly would have to cut the administrative bloat. Community governance would have to take place of the career administrators and corporate consultants. But we also need to consider what we would do with that governance. How would we imagine better ways to do our jobs? What will matter to us when times get hard? What do we want to save and preserve for future generations?

We will likely continue to teach and do mathematical research, in some form, for the foreseeable future. Both education and creativity are basic human needs. We will not give up on them easily. We do, however, need to think about which parts of our jobs are less important and could be discarded.

A Green New Deal for universities? Faculty numbers, especially the numbers of tenured and tenure track faculty, have no relation to how much work actually needs to be done at universities. Our workloads have long been ballooning out of control. New responsibilities are added almost every day. At the same time, faculty positions continue to be eliminated or converted to temporary ones, so that the increasing total workload is shared between fewer faculty. Technology does not solve the problem: as the failed MOOC experiments have shown, small groups and personal contact are at least as important in online teaching as they are in face to face classes.

What if we reversed that? We could hire more people and redistribute the workloads. It would create new jobs, we would all have more time and capacity to have a life outside of work, and the quality of the work we do would likely improve if we did not have to rush it. It would also mean a redistribution of salaries, but I would gladly accept lower pay as a fair price for the rest of the bargain.

Less gatekeeping, more redistribution. We spend an astounding amount of time and energy on gatekeeping: refereeing, proposal evaluations, ranking decisions, writing and reading recommendation letters, editorial work. What if we did not have to do that? Or, at least, if we could reduce it by half or more?

Obviously, gatekeeping would be less intense if the stakes were not as high. If we discontinued the current Hunger Games model where only a handful of decent jobs is available and everyone else is an adjunct with no job security, then perhaps thousands of person-hours would not have to go into the countless meetings, letters and evaluations that are currently mandated every time someone moves one step up the ladder. Some funding applications come with so much administrative work, in both preparation and adjudication, that any funding awarded in the end would not even come close to balancing the cost of that at a fair hourly rate.

Less output, but make it count. We need to stop measuring the quality of the researcher by the quantity of their output and level of their activity. “Productivity measures” such as citation index or counting the number of published papers are deeply

flawed. We like to say that these are imposed on us by administrators. But what would we do if no administrators were there to constrain us? How much of this is actually peer pressure? We should in particular be more realistic about what can legitimately be expected from junior job candidates.

How many conferences and institute programs do we really need? How do we best use them? Even if air travel becomes easier again in the future, it will still have high carbon footprint and will not be great for our health. Online conferences are more environment-friendly, but even so, what if we treated them as actual communication channels instead of markers of prestige and importance? There are other modes of communication and dissemination, such as blogs and social media; what if we put more effort into diversifying and refining those instead of just counting the number of conference appearances, online or otherwise?

"Online" is not the answer to everything. Covid has forced our professional seminars and conferences to go online. This is being presented to us as the environmentally-friendly alternative, since no air travel is required. Similarly, we once thought that email and electronic record-keeping would reduce the amount of paperwork, and time spent on it, through the elimination of paper. We had also overestimated the anticipated energy savings from replacing incandescent lightbulbs with LED lights, not taking into account that when lights are cheaper to use, people install more of them and leave them on for longer. The same could happen with online conferences. Easy availability breeds proliferation. The energy footprint of the internet is not small: Bitcoin mining has been estimated to consume about the same amount of electricity as small countries, and similar estimates for videoconferencing under the Covid regime are likely forthcoming. Online or not, we will have to choose our activities carefully and know where to stop.

Preservation of knowledge. Do we still have time to read other people's papers? 30-40 years ago, people would rediscover previously known results because research dissemination was less effective. (There was no internet, access to professional journals was more limited, preprint servers did not exist.) Now, this happens because young mathematicians are under so much pressure to produce new results, write them up and move on, that they have no time left for reading. It can also happen because papers written hastily are very difficult to decipher even for experts, or because the sheer volume of incoming preprints and publications is too overwhelming.

Knowledge can and does get lost, especially during major upheavals. We need to spend less time "producing" new papers making incremental progress, and pay more attention to consolidation, exposition and preservation of the knowledge we already have.

Equity, social justice, and collective action. Less stratified fields, with less gatekeeping, are usually good for diversity and equity. What is less appreciated is that this works in both directions. Feminist, anti-racist, and social justice groups have developed professional norms and codes of conduct that reduce gatekeeping, improve the working climate, and promote cooperation. We can learn from them. I have been drawing on that experience in my own mathematical practice, with good effects.

We need to listen to those who have experience living with scarcity and uncertainty. We need redistribution, badly. We need more equality, less competition, more

cooperation. When it comes to sustainability in particular, we need to listen to Indigenous activists. We have been underestimating their traditional knowledge for too long. They are already on the front lines of defending the environment, doing the hard work for us. We have to learn to work, not just with them, but under their direction.

To make any of this happen, we will need collective action on a scale previously not seen at universities. We will need not only unions, but also coordinated action between them, such as the Bargaining for the Common Good coalition at Rutgers [**TW**]. This, too, is in a feedback loop with equity and redistribution. If we, the tenured faculty, treat our staff, adjuncts and graduate students less than fairly, we might have little luck telling them that "we are all in this together" next time we need their help.

Change will be forced on us. We will have to adapt, one way or another. It's up to us whether we make the transition humane and how much of human knowledge we manage to preserve. We cannot buy our way out of the climate emergency. Capitalism will not save us. Universities, as non-profit organizations dedicated to the pursuit and dissemination of knowledge, should be leading the way. We should experiment and then model the change for others.

We will need to learn to make do with less. We like to say that mathematics only requires a pen and pencil. We may be tested on that.

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